Opportunities for Ubiquitous Sensing in Future Infrastructure

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Outline

Goal is Safe and Sustainable Infrastructure

1. Grand Challenge of Future Urbanization

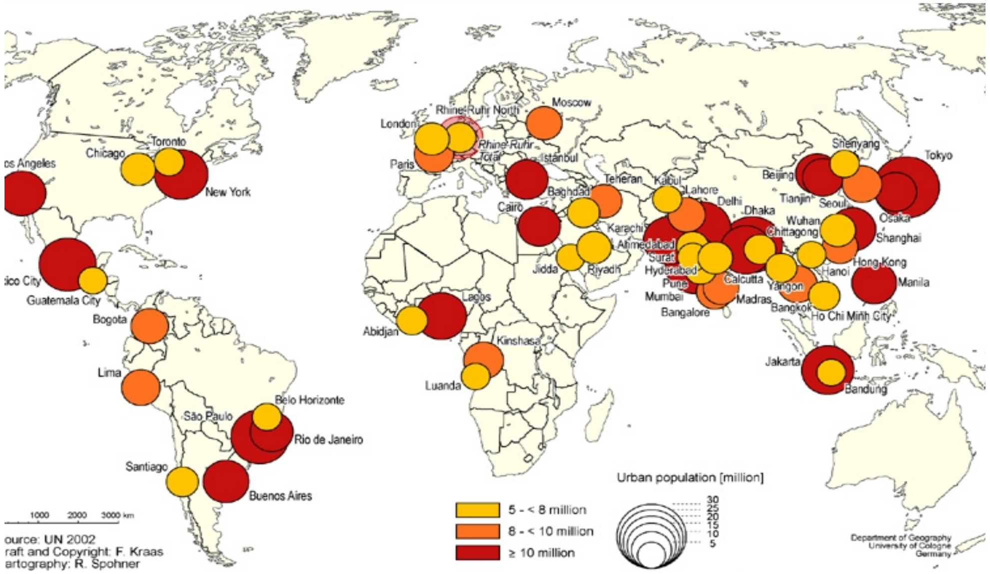
2. Large-scale Long-Term Wireless Monitoring

3. Ad-hoc Interaction for Vehicle-Bridge Interaction

4. Future Outlook: Needs and Opportunities

Rapid Urbanization

- **Rapid emergence of mega-cities (urban populations > 10M):**
  - Rapid advances in agriculture encourages rural populations to move toward cities where economic opportunities are perceived to exist
  - In 2010 significant tipping point achieved with half of the world's population now residing in urban centers (estimated as 2/3 by 2050)
  - 10% of the world's population now lives in 20 mega-cities



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Emergence of Mega-cities

Urbanization Paradox

- **Urbanization holds a very powerful paradox:**
  - Densification of cities leads to economic efficiencies through economies of scale
  - Negative footprint of built environment amortized over increasing populations leads to more sustainable built environments
  - *But, dense urban centers likely lack sufficient resiliency – a grand challenge for civil engineers in the 21st century*

Vulnerability to Natural Hazards

Unsustainable Energy Usage

Accelerated Deterioration

Vulnerability to Natural Hazards

- **Concentration of vulnerability to natural hazards:**
  - Climate change leading to storms of increasing severity
  - Points of interdependency and interconnectivity of infrastructure means greater likelihoods of cascading failures
  - Complexity of handling human and social response to disasters grows faster than at a linear rate with population size

Hurricane Katrina (2005)
Failure of levees lead to cascading failures

Wenchuan Earthquake (2008)
70,000 fatalities and 5 million homeless

Aging Infrastructure

- **Greater user demand from growing populations translates into accelerating aging and deterioration:**
  - Developed nations seeing serious aging in post-WWII infrastructure:
  - Funding model for upkeep is broken in countries like the U.S.
  - Developing nations may see earlier deterioration due to poor construction quality and heavy usage by overloaded vehicles

I-95 Overpass Collapse (1983)
Corrosion beneath pin hanger leads to fatigue

Wuyishan Gongguan Bridge, Fujian (2011)
Bridge collapsed after only 12 years of service

Sustainability

- **As urban centers continue to grow, energy consumption and associated negative environmental impacts grow in tandem:**
  - Concrete is 2nd most used material in the world (2nd only to water):
  - Approx. 7-10% of GHG produced annually from concrete
  - Repeated repair of brittle concrete represents unsustainable practice

Concrete is inherently a Brittle Material
2 metric tons per person per year

About 40% of Life Cycle Energy for Bridges is
Related to Repeated Deck Repairs

Partial Solution: Ubiquitous Sensing

- **Major advances in sensors made over the last 25 years**
- **Sensor data for the assessment of performance and health:**
  - More cost-effective management of aging infrastructure
  - Condition-based maintenance and longer-service lives translate into higher amortization of initial carbon footprint
- **Fusion with sensor data from natural hazard domain:**
  - Ground motion arrays provide a measure of seismic demand
  - Weather data from meteorological observatories understand storms

The proverbial “double-edge” sword

Challenges of Ubiquitous Sensing

- **The high cost of sensing technology remains a challenge:**
  - Many of the sensors available come with high procurement costs
  - With wired monitoring systems prevalent, high installation costs on the order of a few thousand dollars per sensing channel
- **Data inundation is the primary bottleneck:**
  - “We swim in an ocean of data yet remain thirsty for information”
  - Lack of scalable data processing tools for health assessment
  - Cost-benefit analyses of monitoring difficult if not impossible

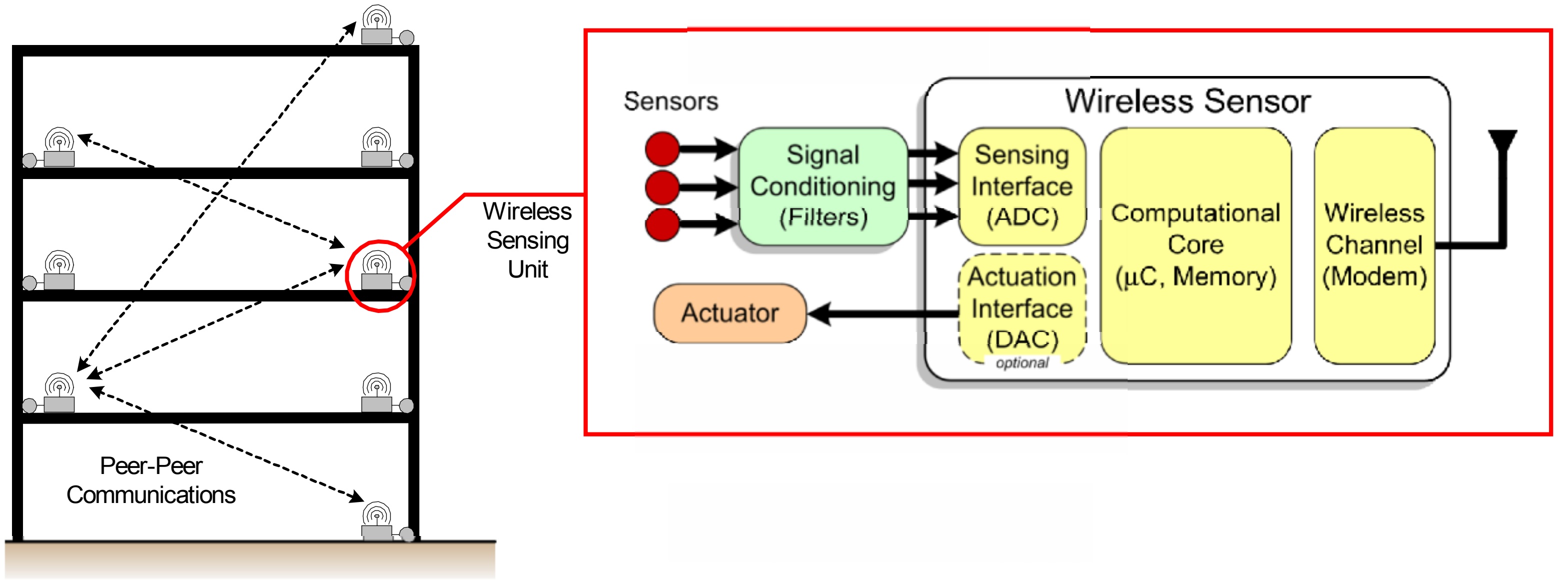
Golden Gate, CA (76 channels)

Tsing Ma Bridge, HK (+300 channels)

Vincent Thomas, CA (26 channels)

Emergence of Wireless Telemetry

- **First proposed at Stanford University in the early 1990's**
- **Wireless telemetry represents a major-paradigm shift:**
  - Wireless communication eradicates need for expensive cabling
  - Ad-hoc connectivity allows for peer-to-peer, ad-hoc communication
  - Computing in node design permits sensor-based data interrogation



Architectural design of wireless structural monitoring systems

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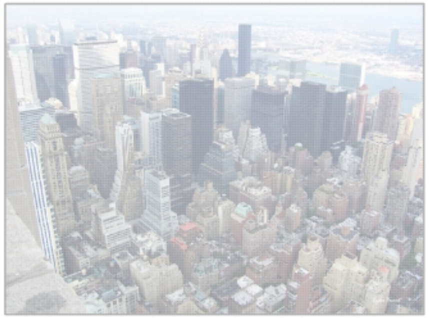
Goal is Safe and Sustainable Infrastructure



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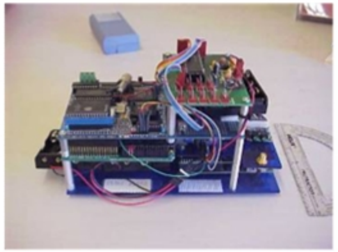
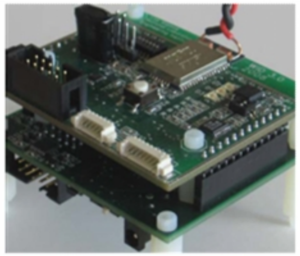
Wireless Sensor Families

Academic wireless sensor prototypes for SHM


Stanford WiMMS (1996)


Stanford WiMMS-II (2003)


Michigan Narada (2005)


LANL WiDAQ (2008)

UC Berkeley "Mote" wireless sensor family for generic applications


Crossbow MoteZ (2002)

Dust Inc Mote (2006)

Intel iMote (Gen1) (2004)


Crossbow iMote (Gen2) (2009)

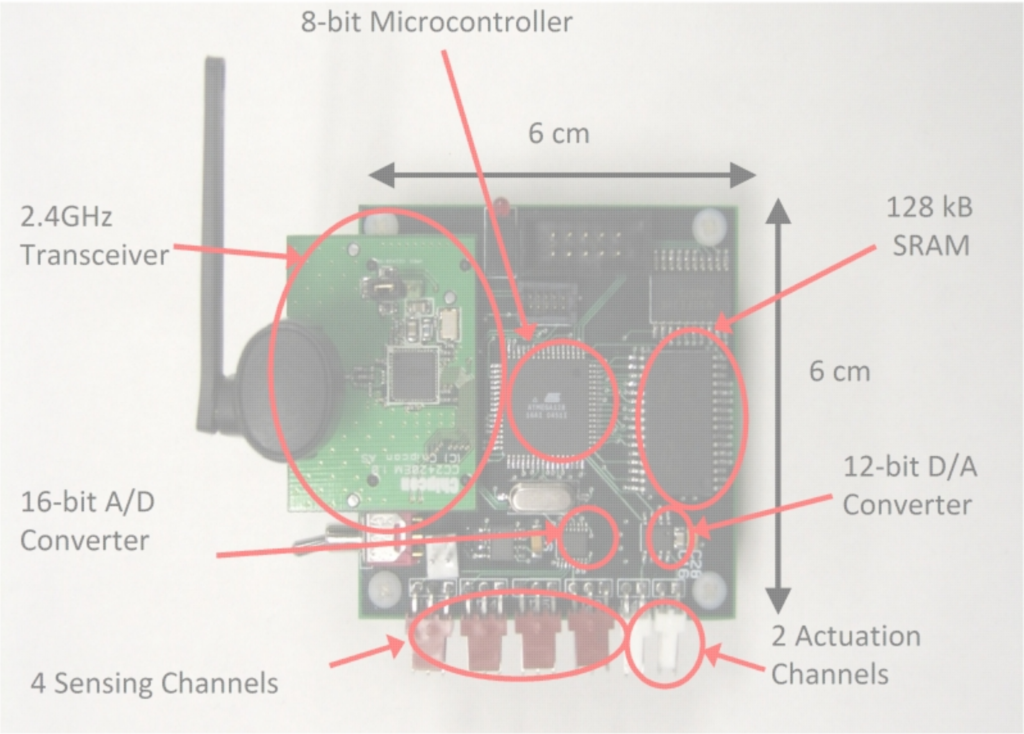
Other commercial wireless sensors for general purpose data acquisition

Microstrain G-Link (2005)


National Instruments (2010)

Narada Wireless Sensor

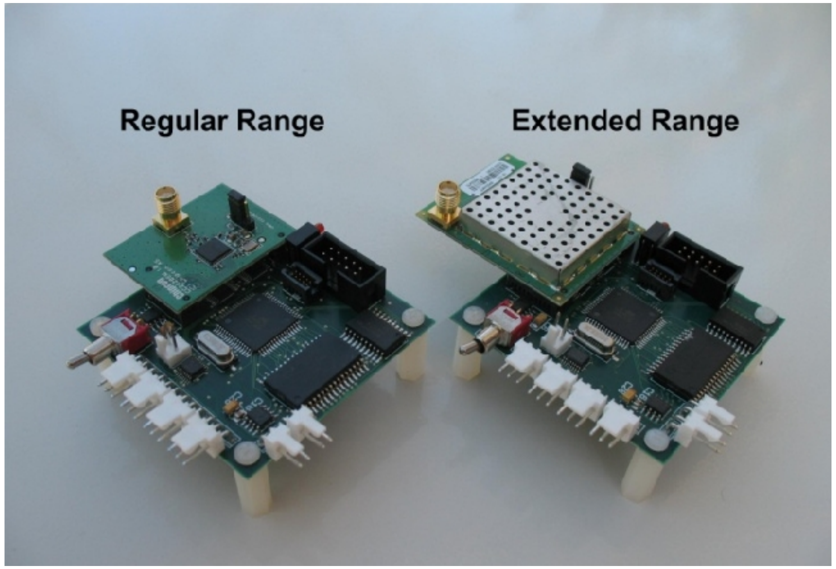
- **Wireless sensor for SHM application (Swartz et al. 2005):**
  - 16-bit ADC resolution on 4 channels capable of high rates (100 kHz)
  - IEEE802.15.4 radio offers interoperability with other sensors
  - Rich embedded processor for sensor-based data interrogation



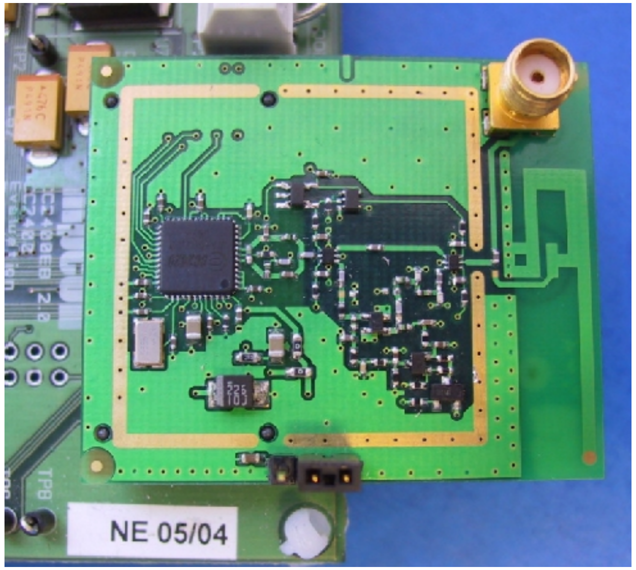
SPECIFICATIONS	
Cost	$175 per unit
Form Factor	5 cm x 6 cm x 2 cm
Energy Source	5 AA Batteries
Active Power	200 mW
Sleep Power	20 mW
Range	100 m
Data Rate	250 Kbps
Sample Rate	100 kHz

Power Amplified Telemetry

- **Large-scale structures require long-range communication**
  - Civil structures, such as bridges, defined by 100’s and 1000’s meters
- **To achieve greater range, *Narada* amplifies its output:**
  - Power amplifier circuit designed to achieve 10 dBm output gain
  - Communication range (line-of-sight) is over 700 meters



Narada with regular and extended range radios



Power amplifier circuit for CC2420

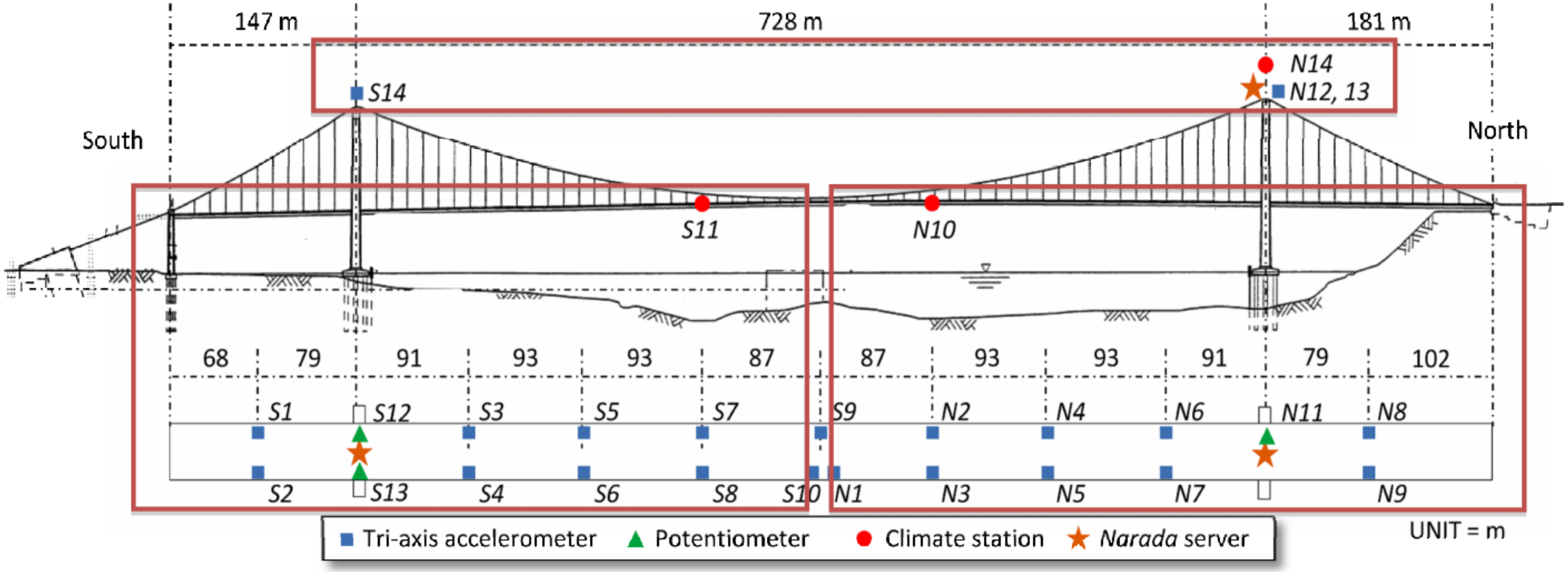
New Carquinez Bridge

- **New Carquinez Bridge (constructed 2003):**
  - Located in the San Francisco Bay Area (Vallejo, CA)
  - Total bridge length is 1056 m (main span of 728 m)
  - Main deck consists of steel orthotropic box girders
  - Hollow concrete tower legs and pre-stressed link beam

New Carquinez Bridge, California

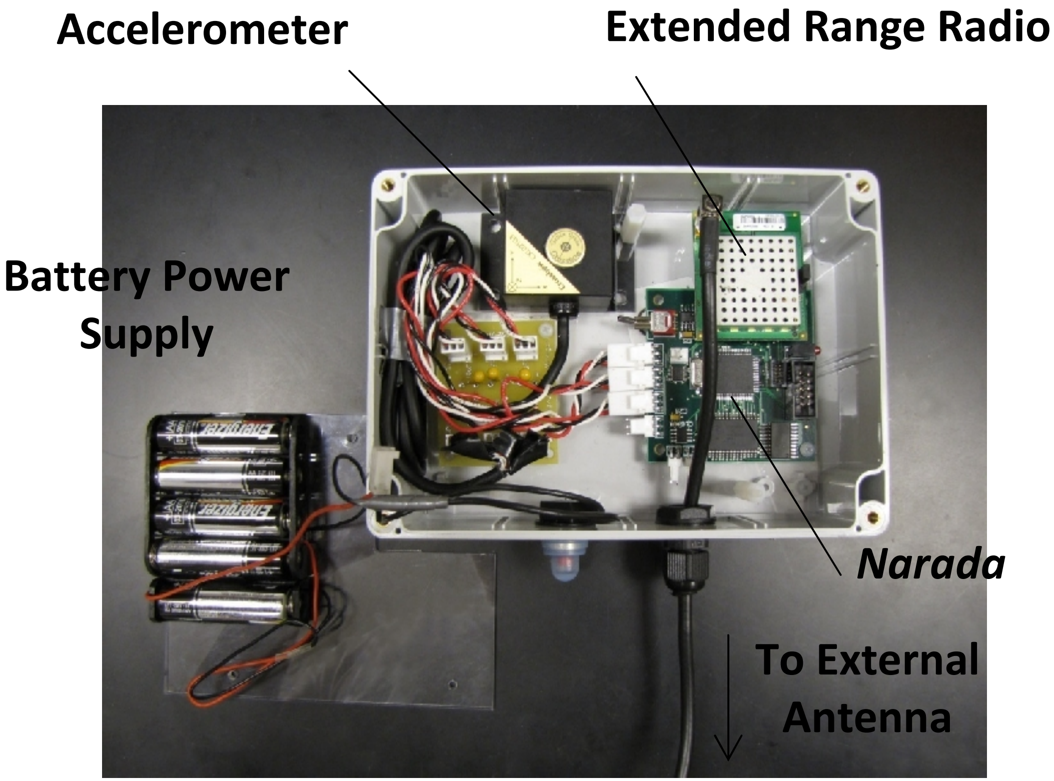
Instrumentation Strategy

- **28 wireless sensor nodes collecting 81 channels (2009):**
  - 19 tri-axial accelerometers measuring main deck
  - 3 tri-axial accelerometers measuring vibrations at tower top
  - Wind vane, anemometer and temperature in three locations
  - 3 string potentiometers to measure deck movement relative to tower

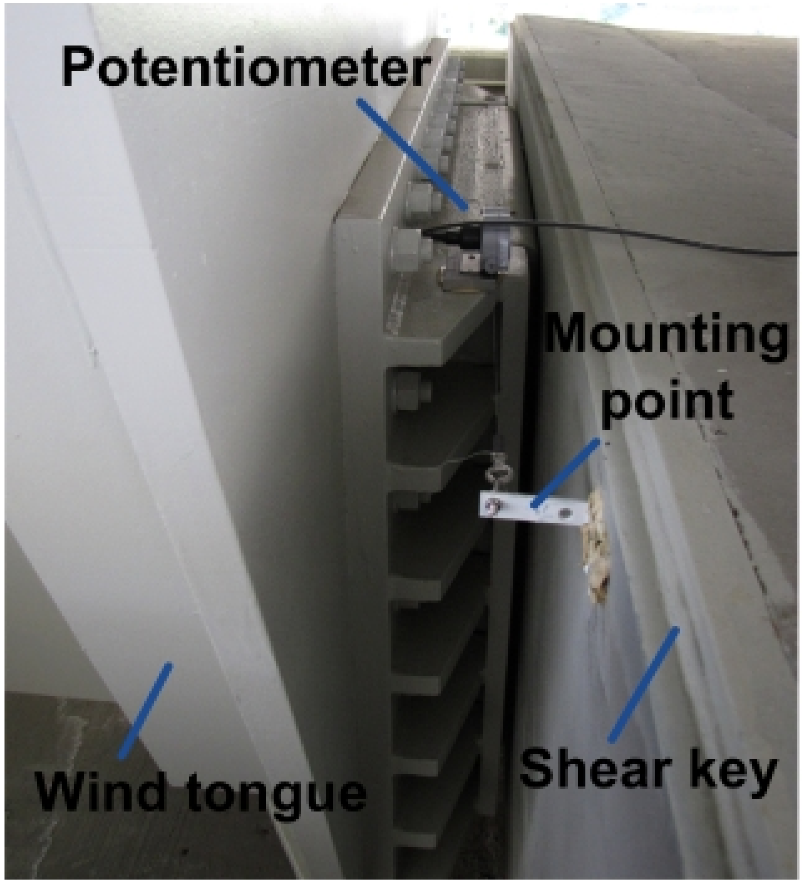


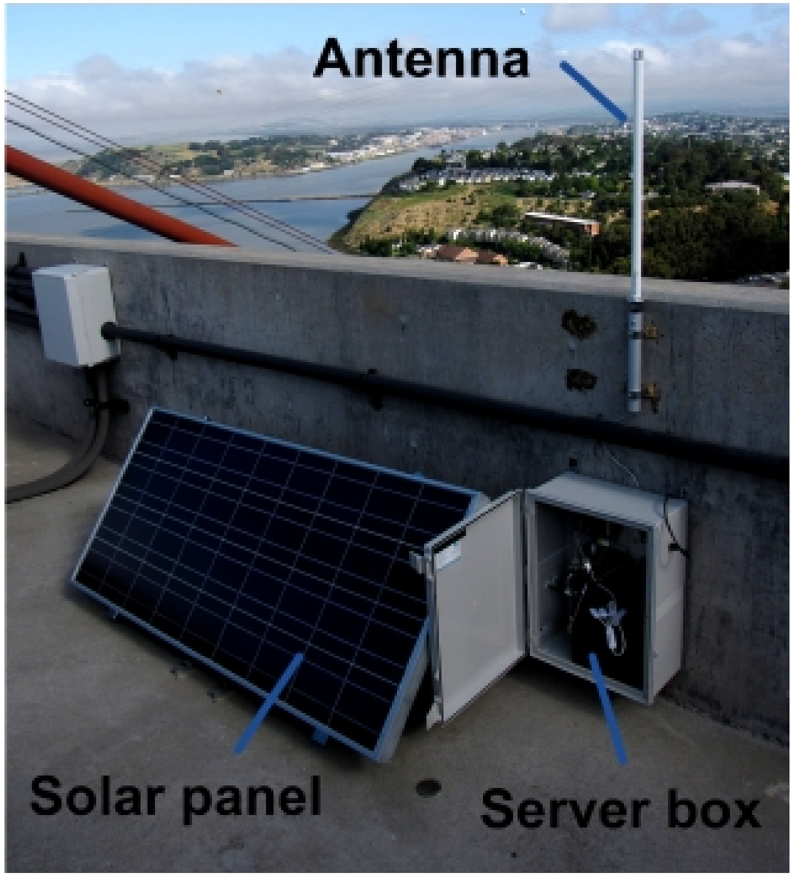
Packaged *Narada* Units

- **Packaging for long-term deployment on NCB:**
  - Water tight enclosure for all electronics
  - Magnetic mounting for quick and easy installations



Installation Details





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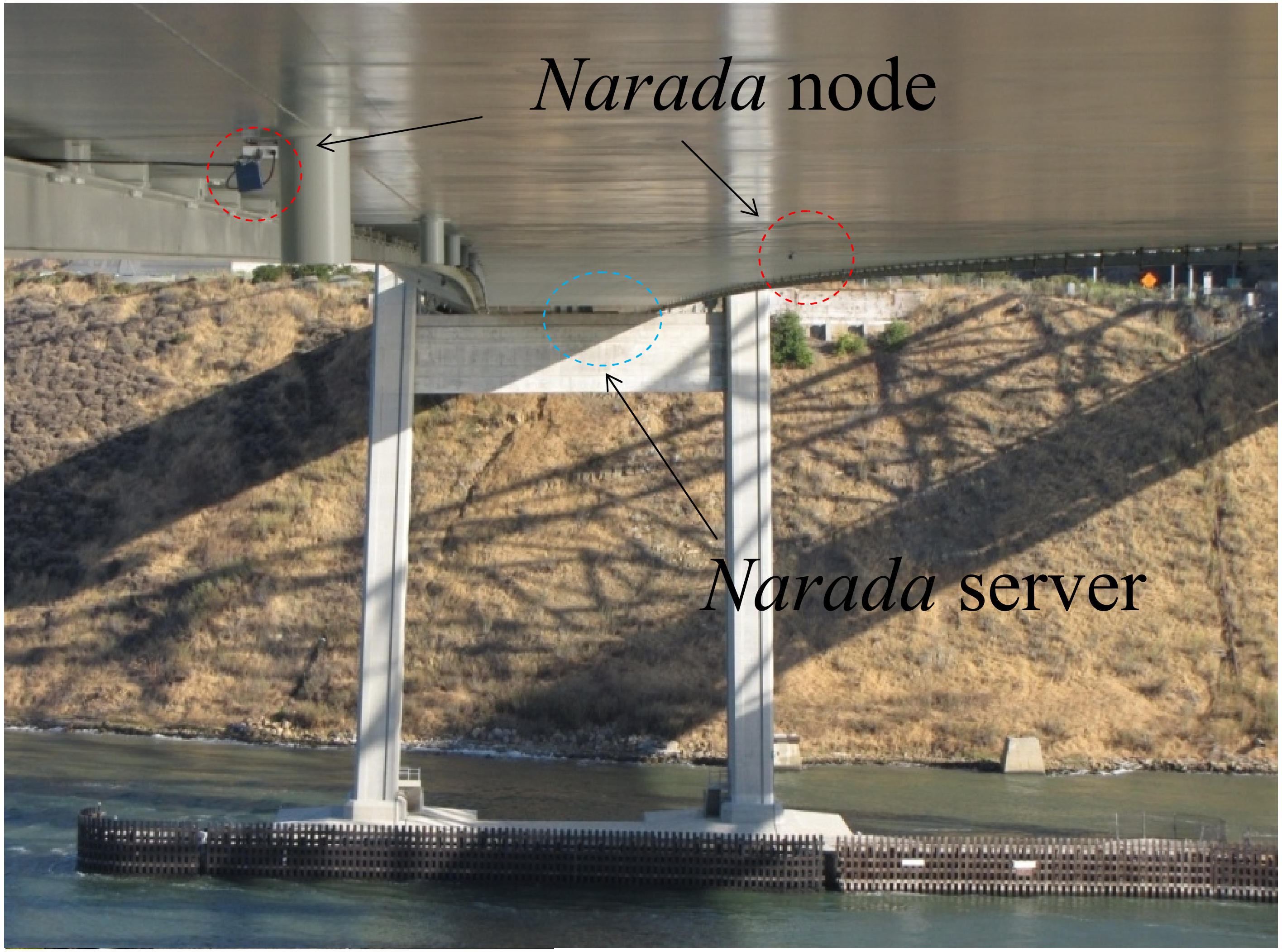
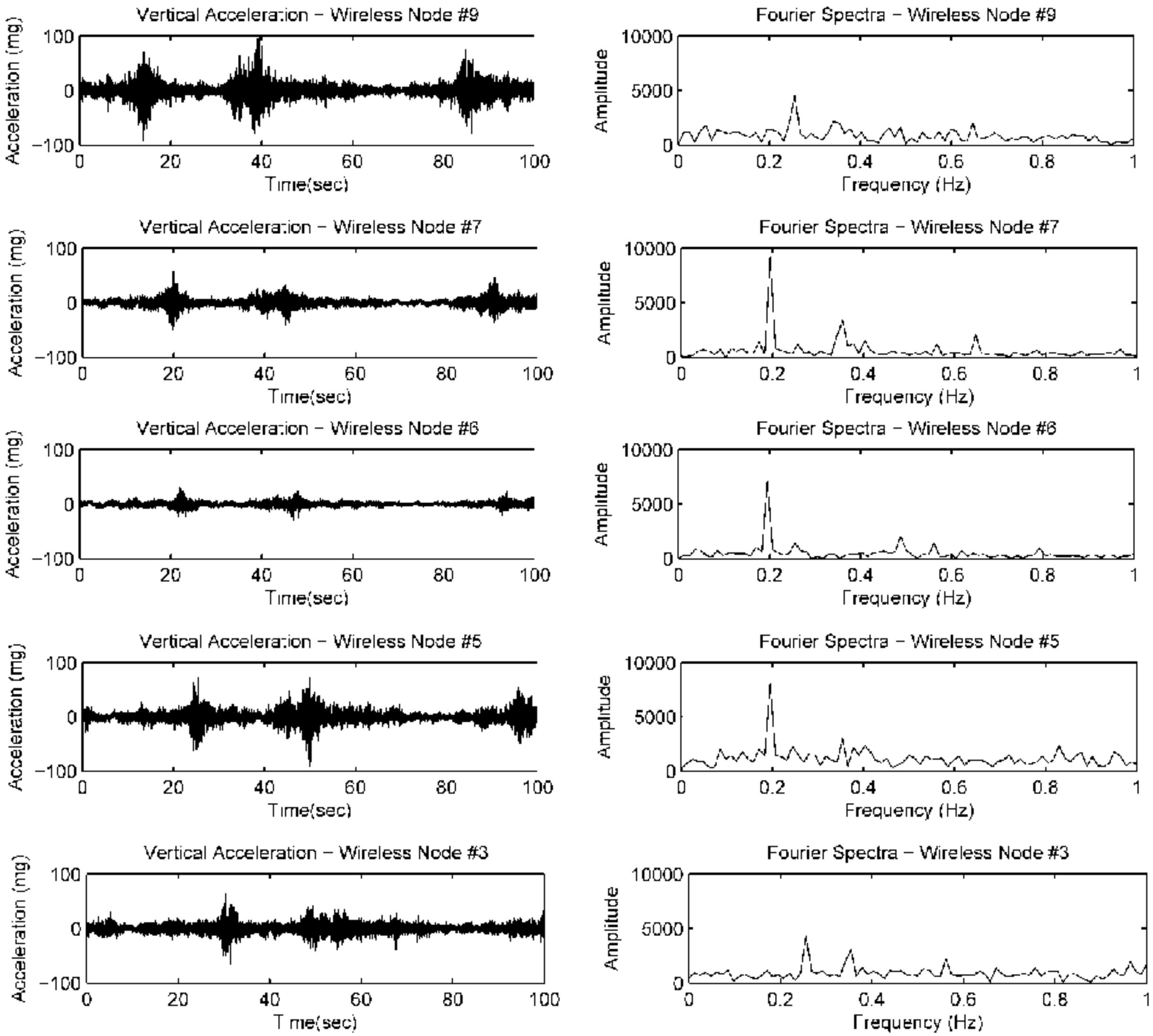
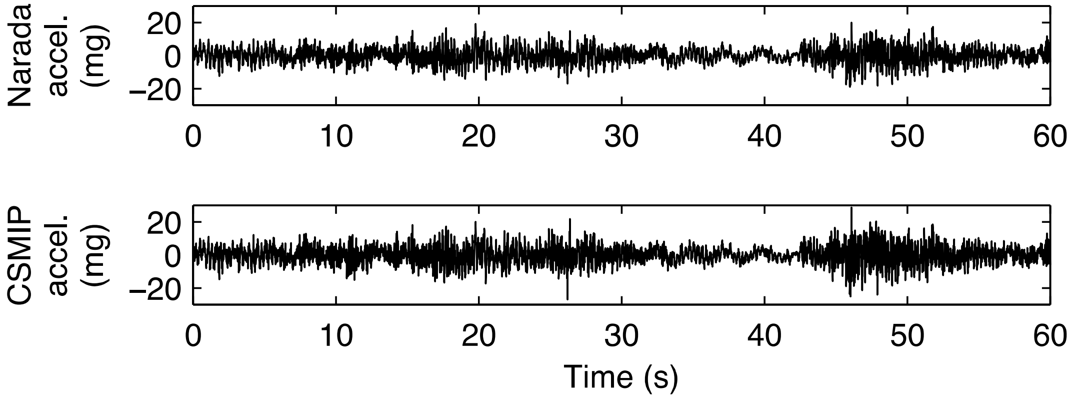


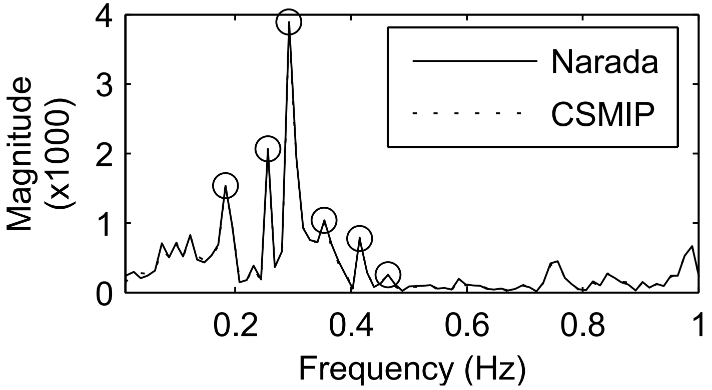
Ambient Vibrations



Comparison to CSMIP Data

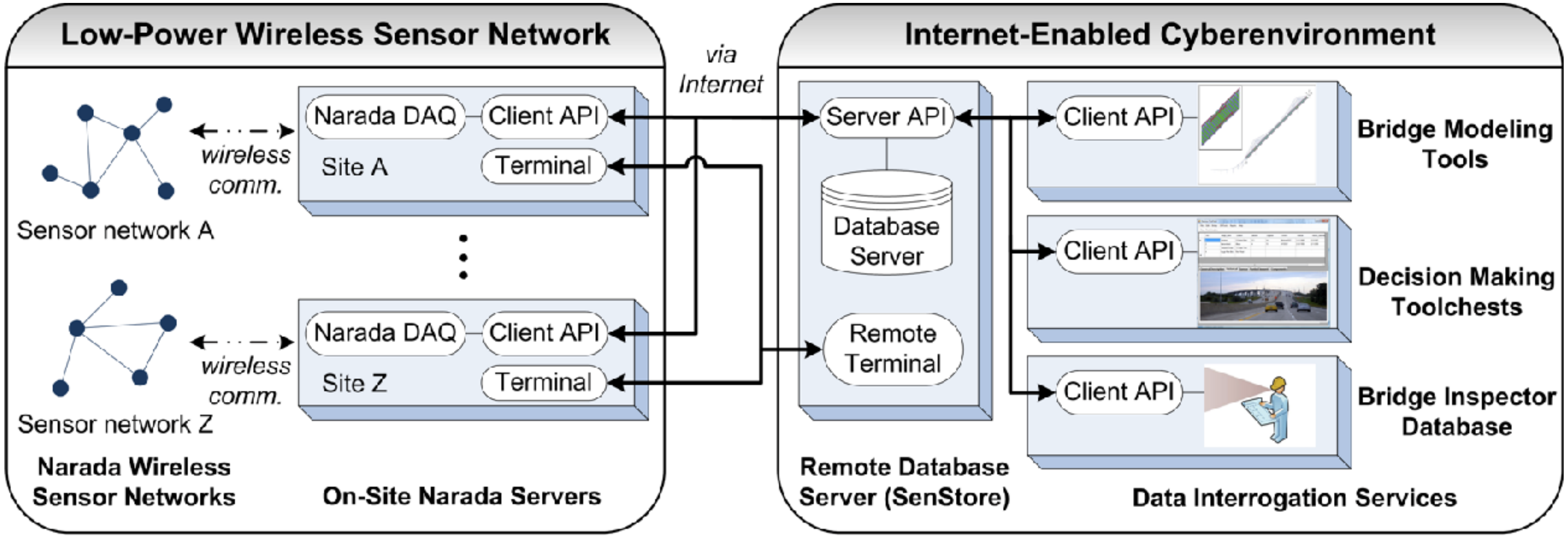
- **California Strong Ground Motion Instrumentation Program:**
  - NCB already has a permanent seismic monitoring system installed





Cyberinfrastructure

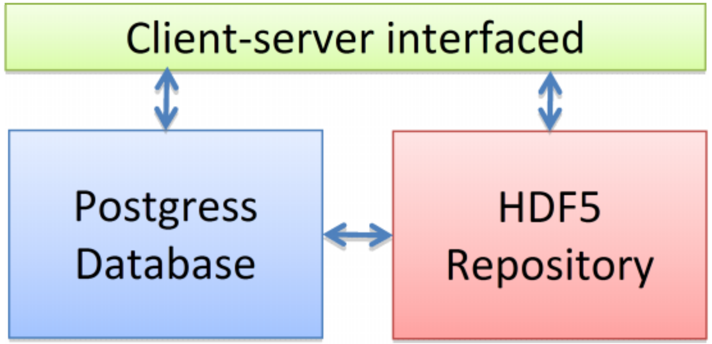
- **What do you do with data from hundreds of channels?**
  - Sensor technology has outpaced data management tools
- **Cyberinfrastructure tools offer enormous potential:**
  - Data combined with powerful analytical tools
  - Physics- and statistics-based information discovery



Proposed Cyberinfrastructure Framework for Bridge SHM

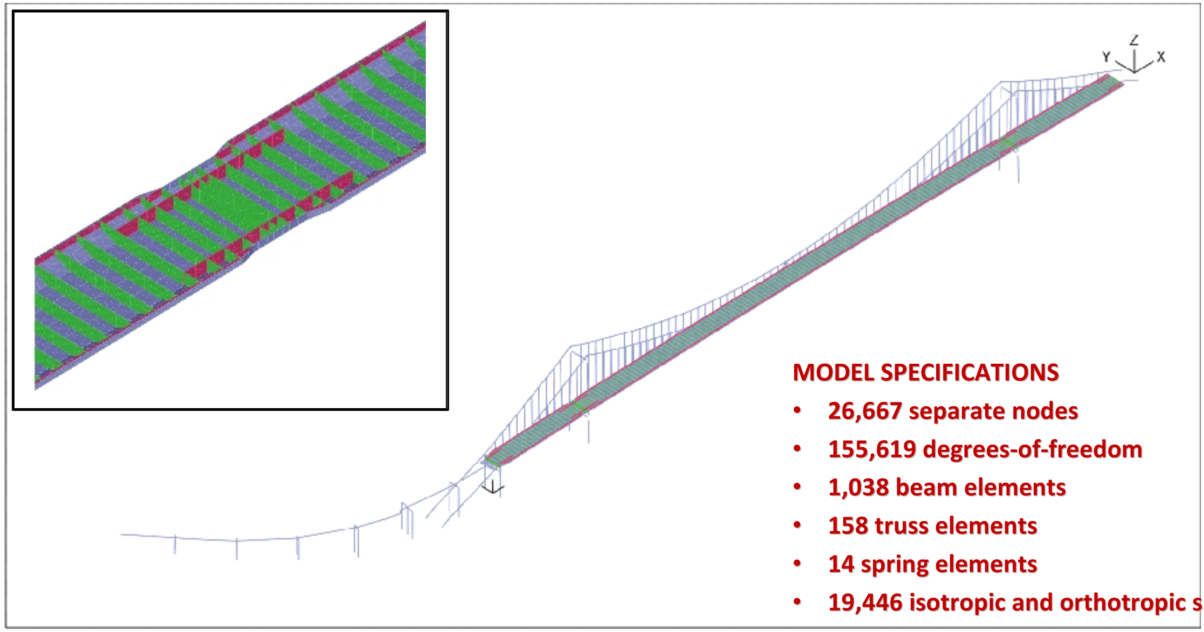
SenStore Database Architecture

- **Relational Database:**
  - Relational database to store all non-sensor bridge information
  - Full description of bridge for automated finite element modeling
  - Bridge management information (inspector reports, etc)
- **HDF5 Repository:**
  - Relational database not an efficient means of storing sensor data
  - Natural means of storing large tracks of time history data
- **Client-server interfaces exposed for data extraction**



Owner Objective: Mode Extraction

- **Owner of bridge (Caltrans) concerned about seismic safety:**
  - Concern is the seismic safety of the bridge during large earthquakes
  - Require high-fidelity models of bridge to simulate seismic behavior
- **Seek modal data for updating of baseline FEM model:**
  - Modal frequencies and mode shapes used to update ADINA model

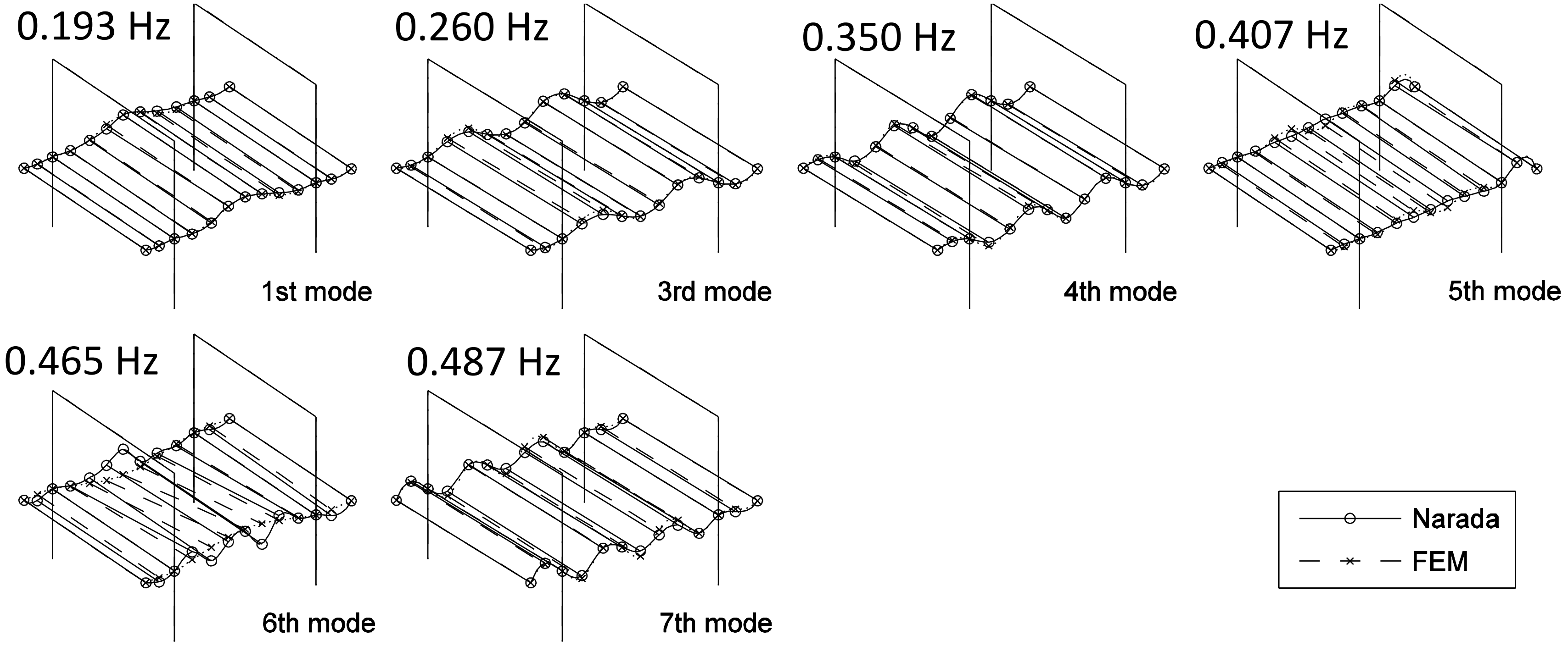


MODEL SPECIFICATIONS

- **26,667 separate nodes**
- **155,619 degrees-of-freedom**
- **1,038 beam elements**
- **158 truss elements**
- **14 spring elements**
- **19,446 isotropic and orthotropic shell elements**

Extracted Mode Shapes

- **In-network estimation by Frequency Domain Decomposition (FDD) mode shape estimation algorithm:**
  - Distributed implementation proposed by Zimmerman *et al.* 2009
  - Excellent agreement with model updated finite element model



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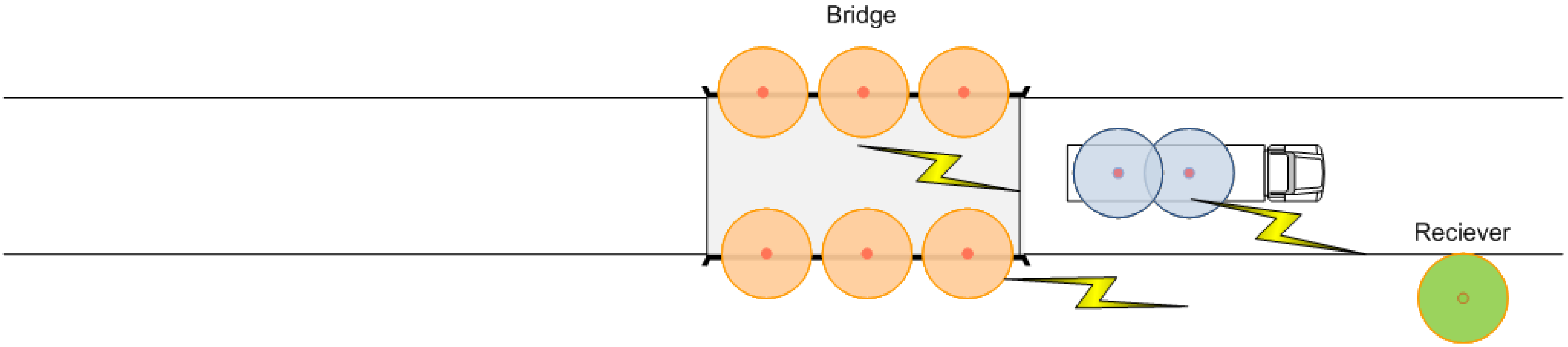
Load Monitoring of Bridges

- **Long-standing challenge of SHM of bridges is the lack of information pertaining to their loads:**
  - Most SHM algorithms to data are output-only (capacity-based)
  - Dire need for an elegant approach to assessing structural demand
- **Heavy trucks represent key load of interest:**
  - Repeated loading by heavy trucks leads to deterioration
  - Highly complex vehicle-bridge interaction problem

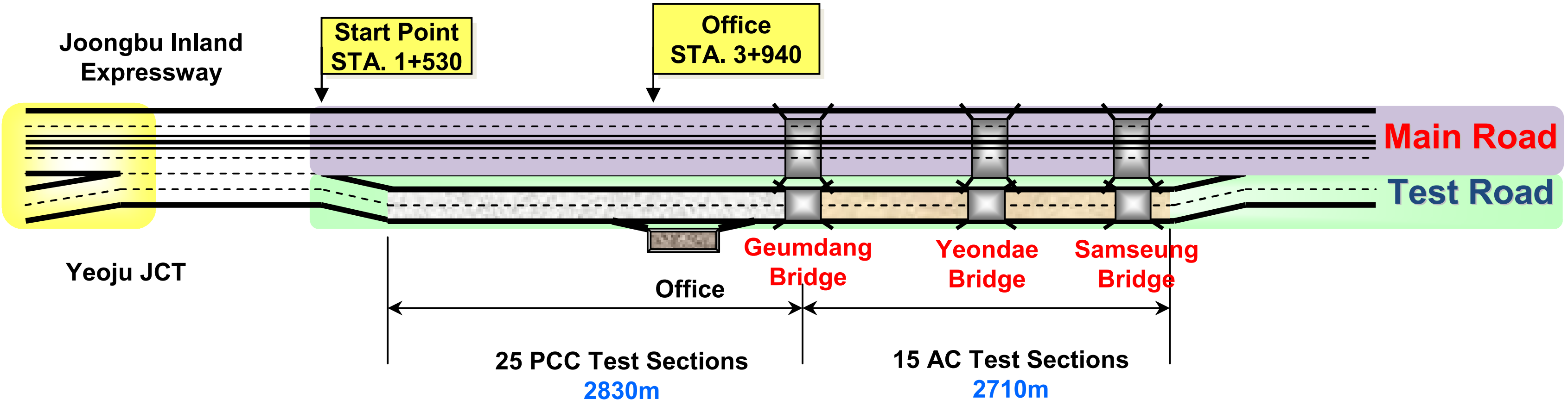
Wireless Load Monitoring

- **Due to the mobility of the moving truck load, wireless sensing is a perfect solution:**
  - Truck sensors can connect in an ad-hoc manner with bridge sensors
  - Time synchronized vehicle and bridge response data collected
- **Truck-based wireless monitoring system:**
  - Vertical accelerometers to measure truck vertical vibration
  - Horizontal accelerometers to measure truck horizontal acceleration
  - Gyroscope sensor to measure truck pitching action



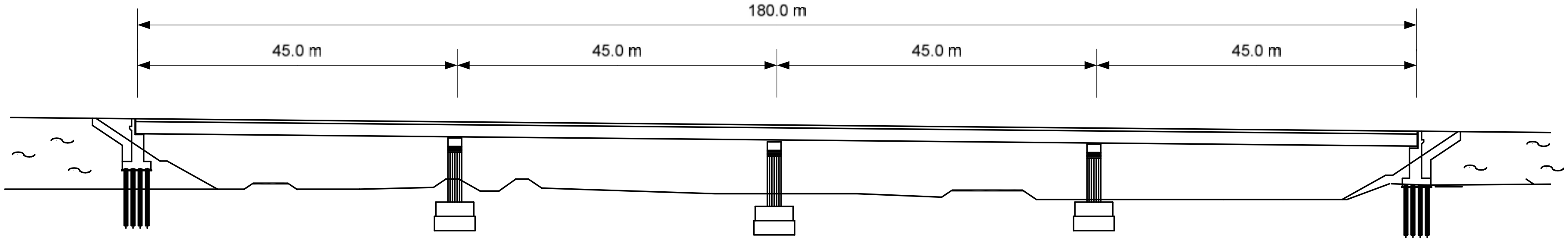
Korea Expressway Testbed

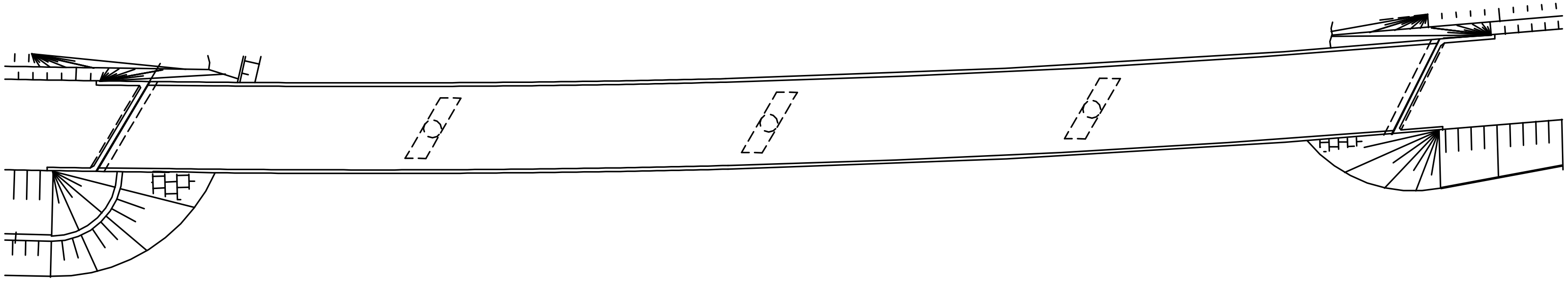
- **International SHM Highway Bridge Testbed Project:**
  - In collaboration with Prof. Chung-Bang Yun (KAIST), Dr. Lee (Korea Expressway Corporation), and Prof. Hoon Sohn (KAIST)
  - Open access for the international SHM research community to test and validate sensing technologies and data processing algorithms
  - Extensive wireless sensing validation since 2004 on the test road bridges through various short-term deployments

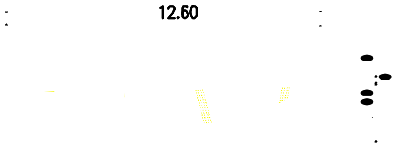


Korea Expressway Corporation Test Road (5.54 km)

Yeondae Bridge (180 m)

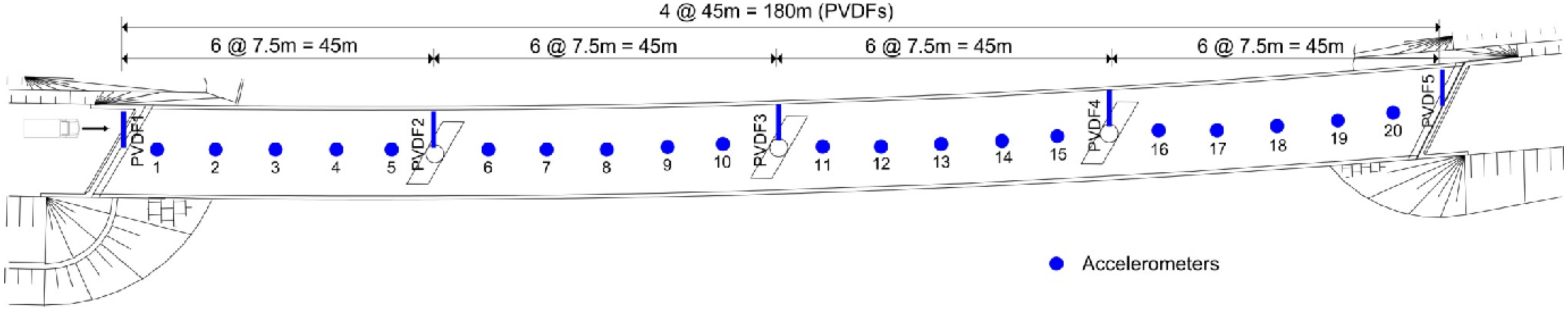






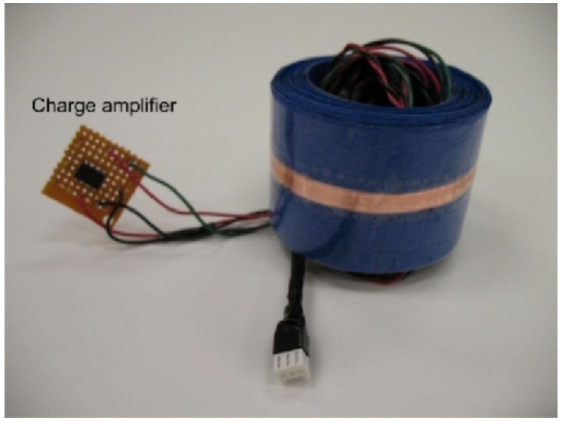
Bridge Instrumentation

- ***Narada*** **wireless monitoring system installed:**
  - 21 vertical accelerometers and 5 PVDF tactile sensor strips



Installed accelerometers

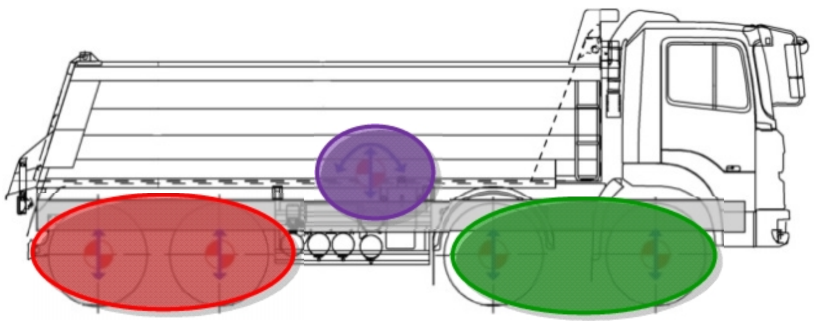
Installed PVDF strips



Tailored PVDF strips

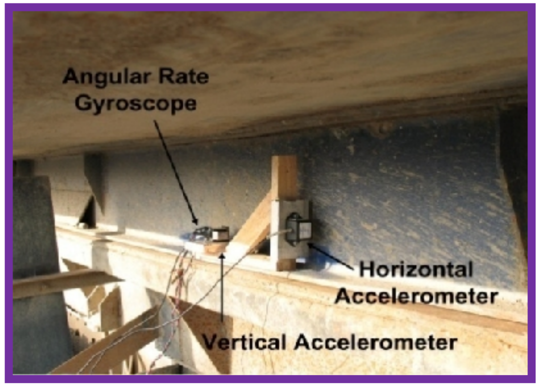
Truck Instrumentation

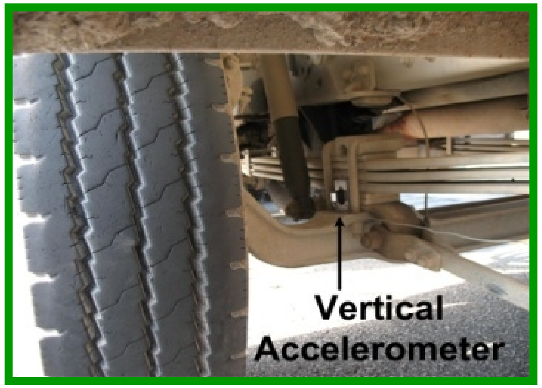
- **Narada wireless sensor network installed in a 21 ton truck:**
  - 6 vertical accelerometers, 1 horizontal accelerometer and 1 gyroscope



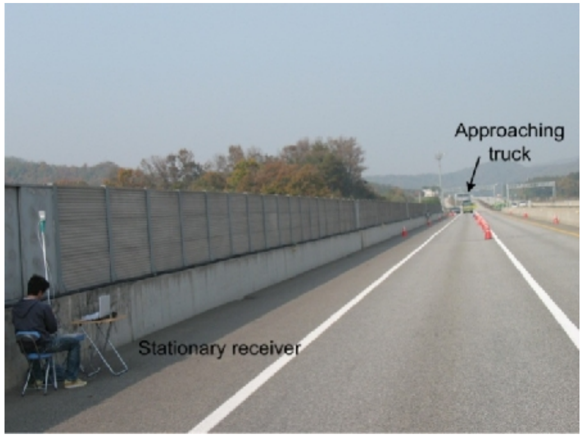
Pitch-plane 6 DOF truck model

Installed 20.9 ton truck

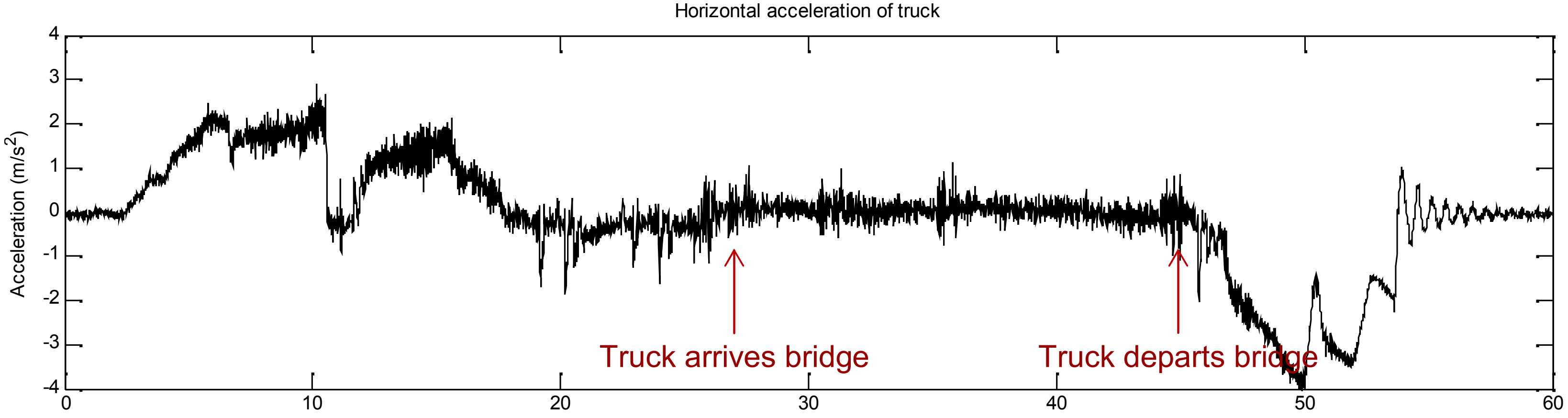


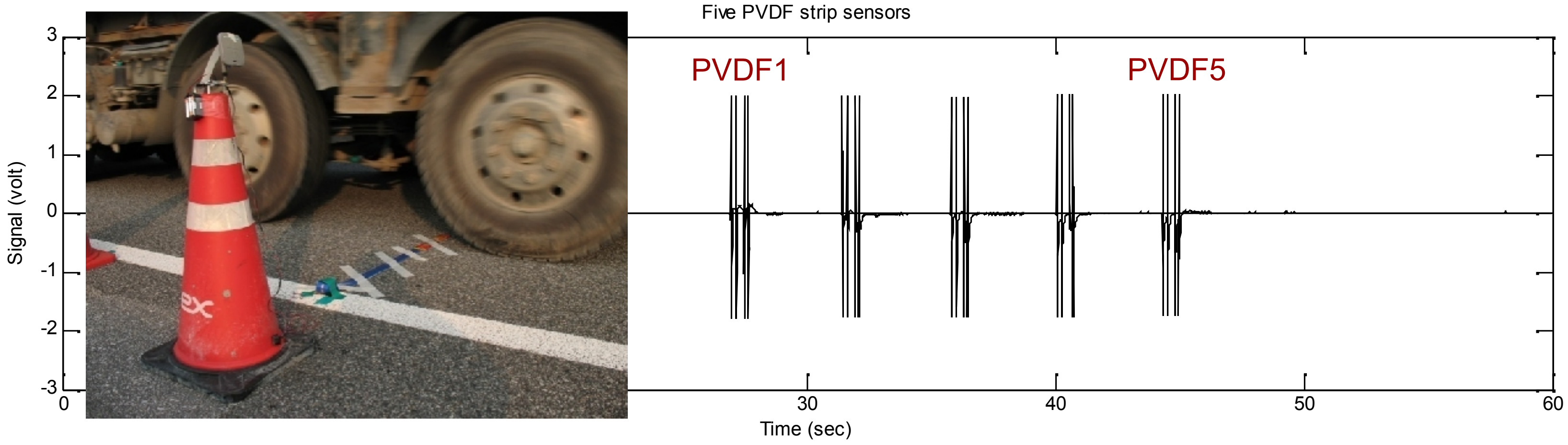


Controlled Dynamic Load Testing



Horizontal Acceleration





Trajectory Estimation

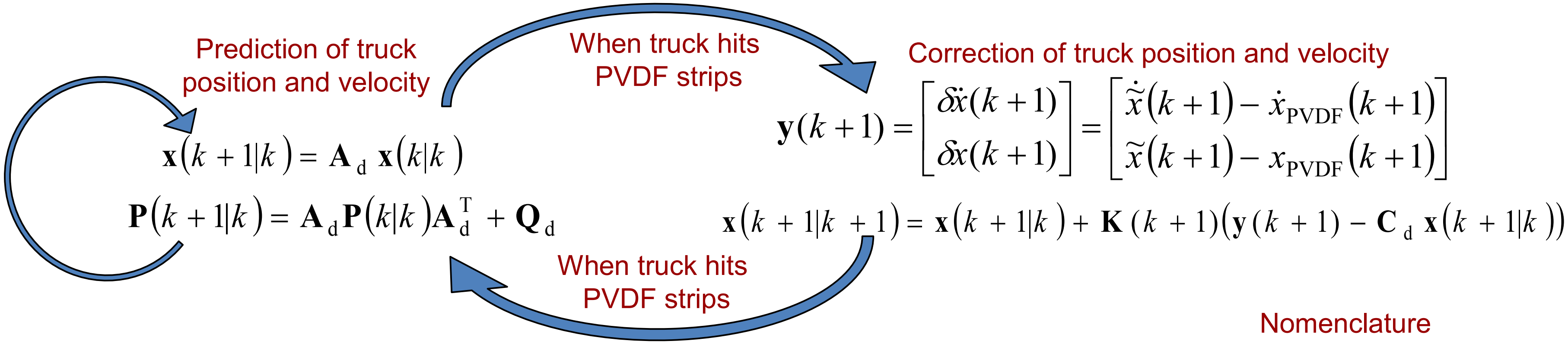
- **Measured acceleration has uncertainty associated with it:**

$$\ddot{x}(t) = a(t) - \delta a(t)$$

$$\delta a(t) = bias(t) + w(t) \longleftarrow$$

Stochastic
$N(0,Q_d)$

- **Truck trajectory estimated based on sensor fusion using a Kalman filtering approach:**



Prediction of truck position and velocity

$$\mathbf{x}(k+1|k) = \mathbf{A}_d\,\mathbf{x}(k|k)$$

$$\mathbf{P}(k+1|k) = \mathbf{A}_d\mathbf{P}(k|k)\mathbf{A}_d^{\mathrm{T}} + \mathbf{Q}_d$$

When truck hits PVDF strips

Correction of truck position and velocity

$$\mathbf{y}(k+1) = \begin{bmatrix} \delta\dot{x}(k+1) \\ \delta x(k+1) \end{bmatrix} = \begin{bmatrix} \dot{\tilde{x}}(k+1) - \dot{x}_{\mathrm{PVDF}}(k+1) \\ \tilde{x}(k+1) - x_{\mathrm{PVDF}}(k+1) \end{bmatrix}$$

$$\mathbf{x}(k+1|k+1) = \mathbf{x}(k+1|k) + \mathbf{K}(k+1)\big(\mathbf{y}(k+1) - \mathbf{C}_d\,\mathbf{x}(k+1|k)\big)$$

When truck hits PVDF strips

$$\mathbf{P}(k+1|k+1) = \big(\mathbf{I} - \mathbf{K}(k+1)\mathbf{C}_d\big)\mathbf{P}(k+1|k)$$

$$\mathbf{K}(k+1) = \mathbf{P}(k+1|k)\mathbf{C}_d^{\mathrm{T}}\big(\mathbf{C}_d\mathbf{P}(k+1|k)\mathbf{C}_d^{\mathrm{T}} + \mathbf{R}_d\big)^{-1}$$

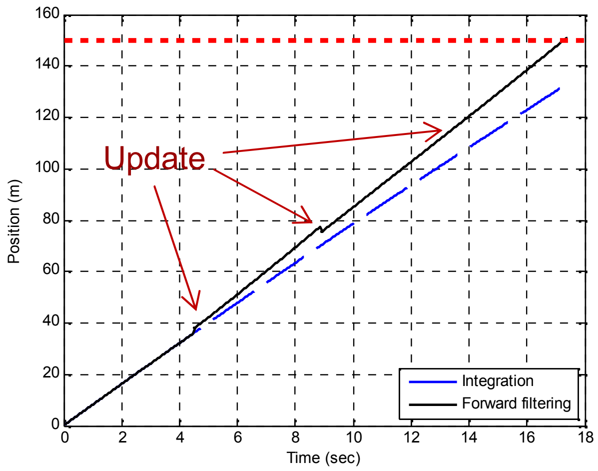
Nomenclature

A_d = integration matrix

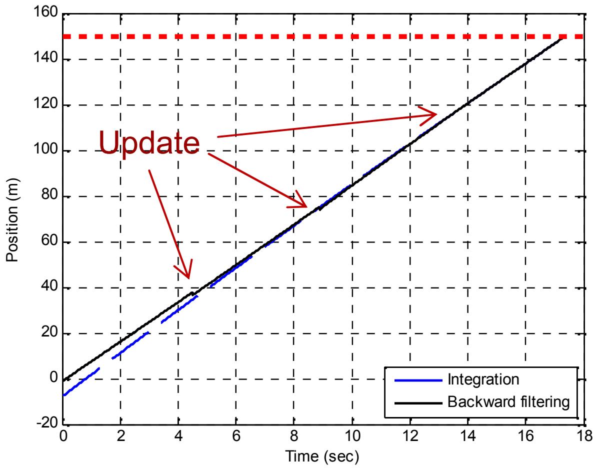
$\mathbf{P}$ = state covariance

$\mathbf{K}$ = Kalman gain

Experimental Verification



Forward Kalman filtering

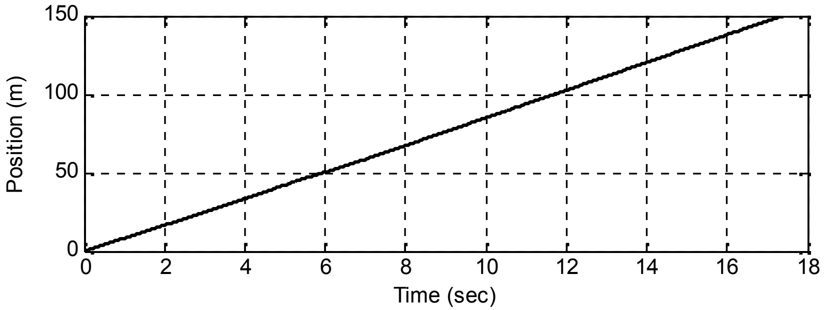


Backward Kalman filtering

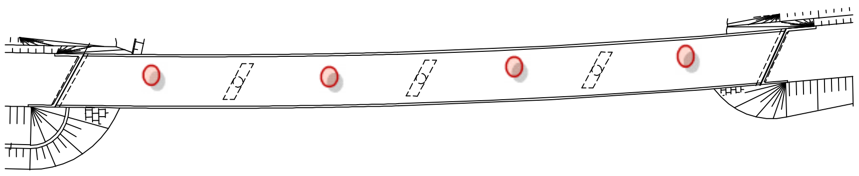
•Fixed-interval smoothing:

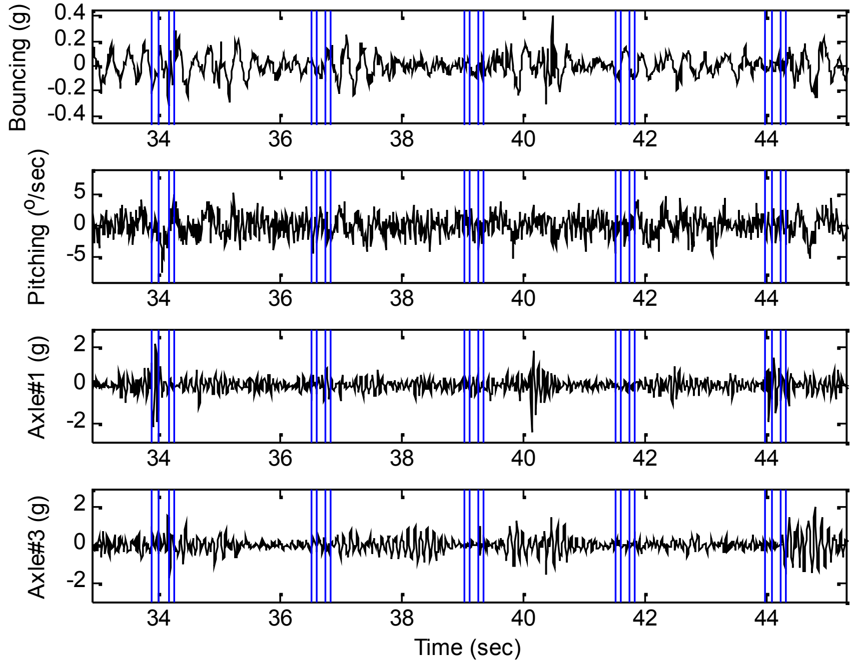
$$\hat{\mathbf{x}}_{\mathrm{s}}(k)=\mathbf{P}_{\mathrm{b}}(k)\left(\mathbf{P}_{\mathrm{f}}(k)+\mathbf{P}_{\mathrm{b}}(k)\right)^{-1}\hat{\mathbf{x}}_{\mathrm{f}}(k)+\mathbf{P}_{\mathrm{f}}(k)\left(\mathbf{P}_{\mathrm{f}}(k)+\mathbf{P}_{\mathrm{b}}(k)\right)^{-1}\hat{\mathbf{x}}_{\mathrm{b}}(k)$$

$$\mathbf{P}_{\mathrm{s}}(k)=\left(\mathbf{P}_{\mathrm{f}}^{-1}(k)+\mathbf{P}_{\mathrm{b}}^{-1}(k)\right)^{-1}$$

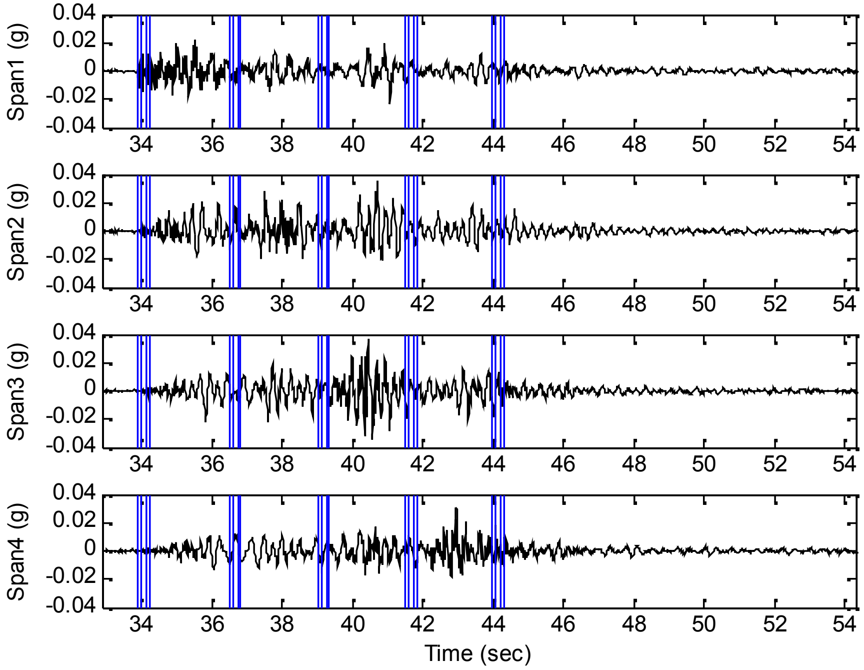


Truck-Bridge Response





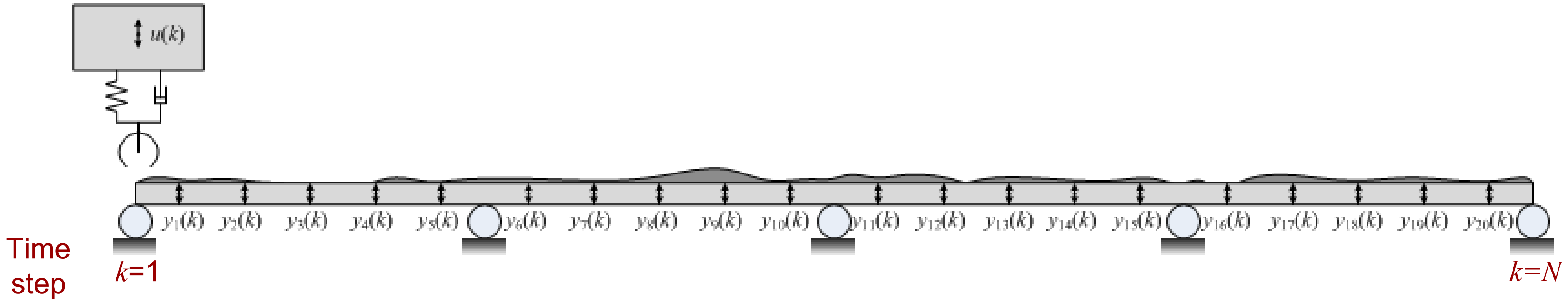
Truck vibration

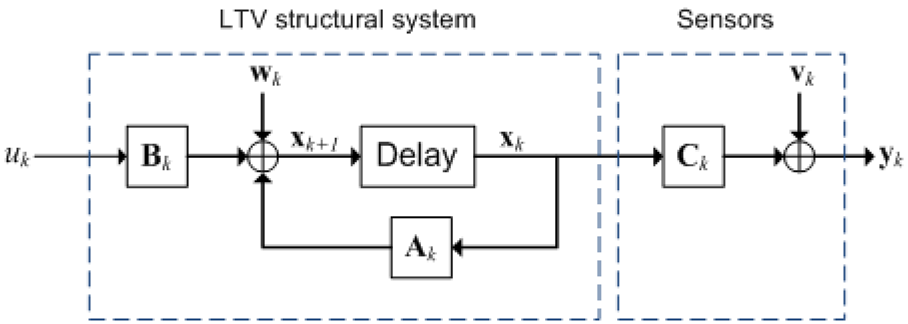


Bridge vibration at span center

Formulation of the Problem

- **Statement of system identification problem:**
  - Linear time-variant (LTV) system
  - Given input and output data, $\{u(k),\, k=1,2,\cdots,N\}$ & $\{\mathbf{y}(k),\, k=1,2,\cdots,N,\cdots\}$
  - Estimate time-invariant $\{\mathbf{A},\mathbf{C}\}$ & time-variant $\{\mathbf{B}(k),\, k=1,2,\cdots,N\}$





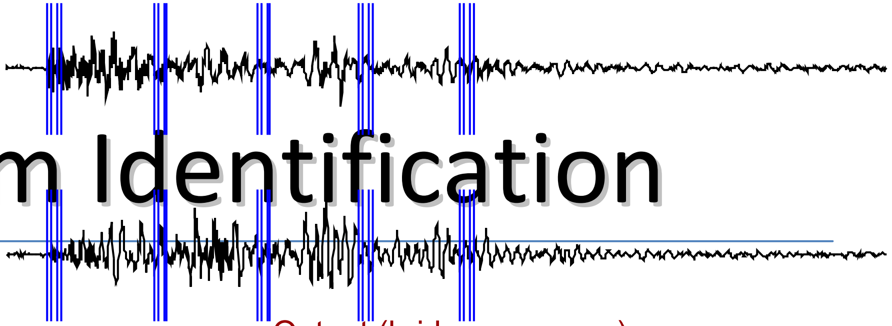
Linear time-variant state-space model:

$$\mathbf{x}(k+1)=\mathbf{A}(k)\mathbf{x}(k)+\mathbf{B}(k)u(k)+\mathbf{w}(k)$$
$$\mathbf{y}(k)=\mathbf{C}(k)\mathbf{x}(k)+\mathbf{v}(k)$$

where $\{\mathbf{A}(k)\in\mathrm{R}^{n\times n},\mathbf{B}(k)\in\mathrm{R}^{n},\mathbf{C}(k)\in\mathrm{R}^{l\times n}\}$

Assuming moderate bridge response:

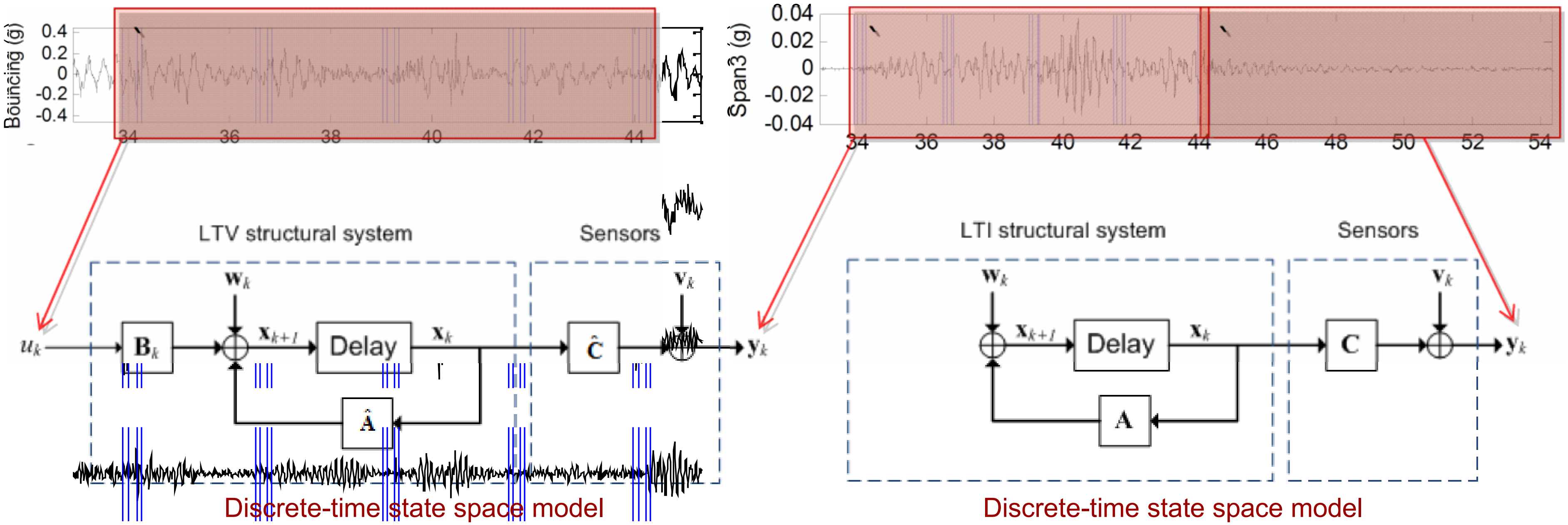
$\{\mathbf{A},\mathbf{B}(k),\mathbf{C},\, k=1,2,\cdots,N\}$



Two-Stage System Identification

Input (truck bouncing motion)

Output (bridge response)



Discrete-time state space model

$$\mathbf{x}(k+1) = \hat{\mathbf{A}}\,\mathbf{x}(k) + \mathbf{B}(k)u(k) + \mathbf{w}(k)$$

$$\mathbf{y}(k) = \hat{\mathbf{C}}\,\mathbf{x}(k) + \mathbf{v}(k)$$

Solution of optimization problem

$$\{\hat{\mathbf{B}}(0), \hat{\mathbf{B}}(1), \cdots, \hat{\mathbf{B}}(N-1)\} = \underset{\{\mathbf{B}(0),\mathbf{B}(1),\cdots,\mathbf{B}(N-1)\}}{\arg\min} \frac{1}{N}\sum_{k=1}^{N} \left\| \mathbf{y}(k) - \hat{\mathbf{y}}(k \mid k-1) \right\|^2$$

Discrete-time state space model

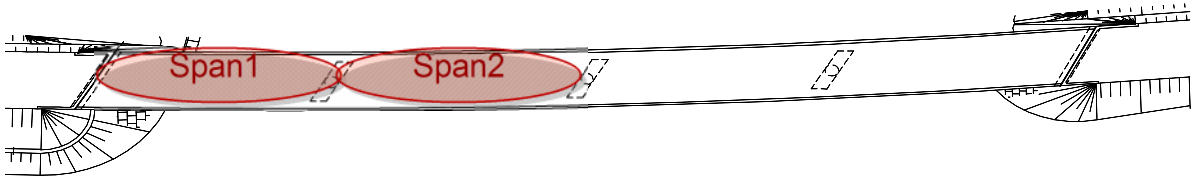
$$\mathbf{x}_{k+1} = \mathbf{A}\,\mathbf{x}_k + \mathbf{w}_k$$

$$\mathbf{y}_k = \mathbf{C}\,\mathbf{x}_k + \mathbf{v}_k$$

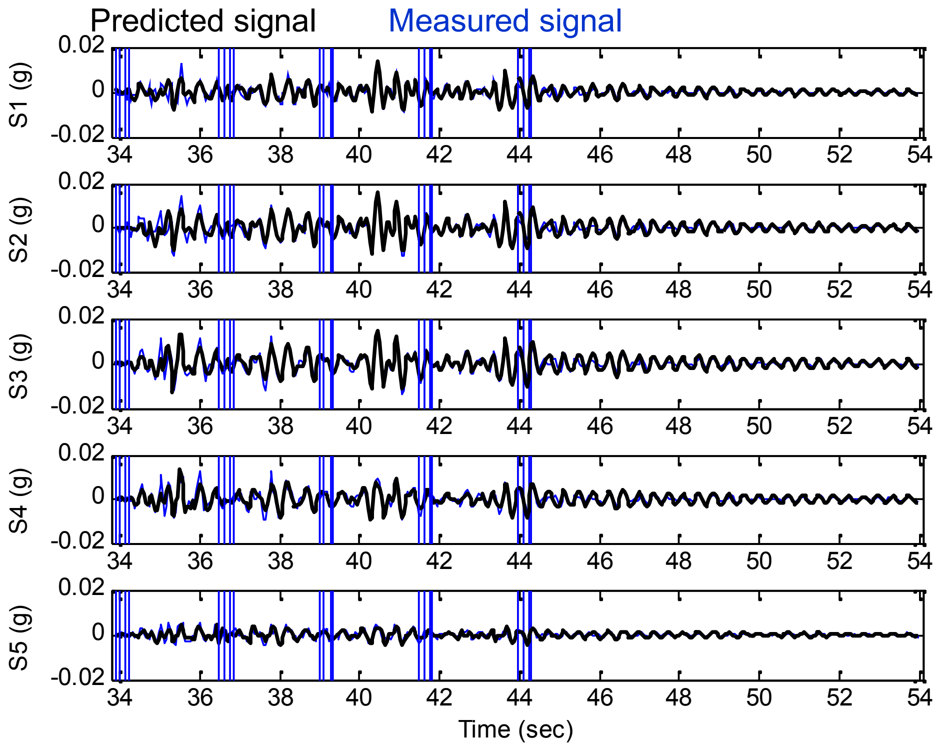
Least squares solution

$$\begin{pmatrix} \hat{\mathbf{A}} \\ \hat{\mathbf{C}} \end{pmatrix} = \begin{pmatrix} \hat{\mathbf{X}}_{i+1} \\ \mathbf{Y}_{i|i} \end{pmatrix} \left(\hat{\mathbf{X}}_i\right)^{\dagger}$$

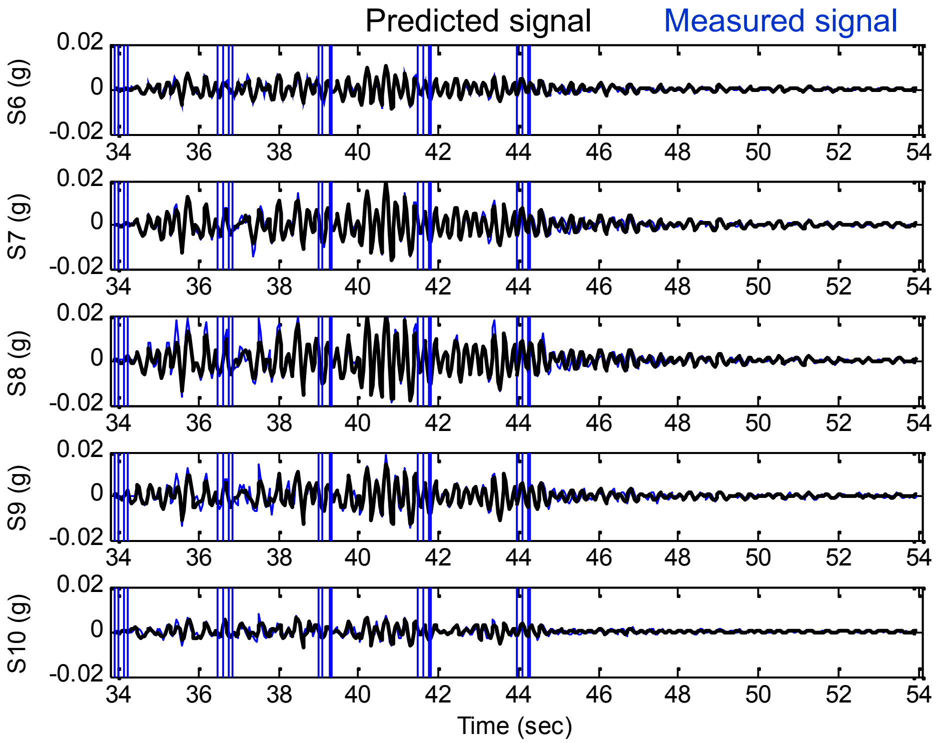
Input-Output Predictions



Span1

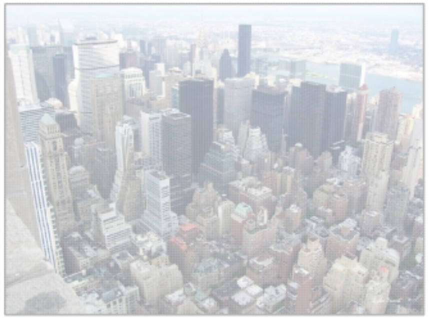


Span2



Outline

Goal is Safe and Sustainable Infrastructure



1. Grand Challenge of Future Urbanization

2. Large-scale Long-Term Wireless Monitoring

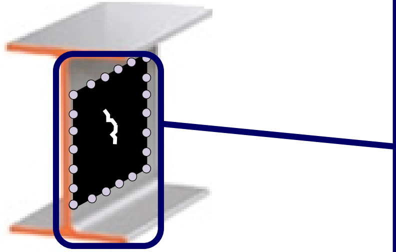
3. Ad-hoc Interaction for Vehicle-Bridge Interaction

4. Future Outlook: Needs and Opportunities

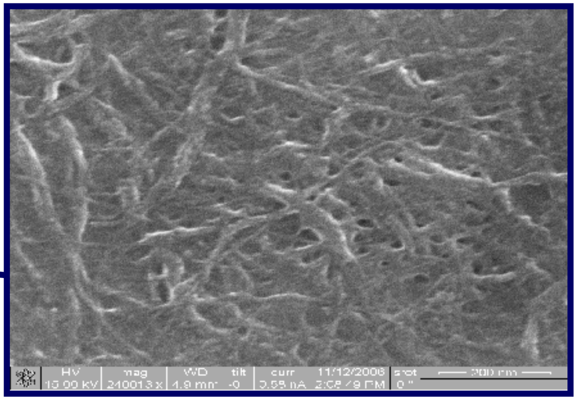
Future Needs: Sensors

- **Resolution of power issues associated with wireless sensors:**
  - Power harvesting in concert with reduction of power consumption
- **Novel types of sensors for SHM:**
  - Sensors that attempt to sensor damage directly
  - Current monitoring paradigm is based on point-based sensors
  - Beginning to see sensors offering distributed mapping

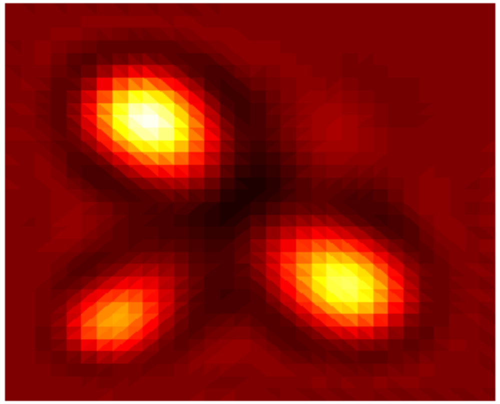
Multi-functional material approach with distributed sensing



Proposed Approach



Self-sensing CNT appliqué (Loh et al. 2008)



Damage map of impact damage (Loh et al. 2008)

Future Needs: Information Extraction

- **Scalable data management system ("middleware"):**
  - Scalable databases for the storage of sensor data but also geometric information, material information, inspection reports, etc.
  - Client-server abstractions for pushing of data into database and secure extraction of data by remote software clients (data analysis tools)
- **Direly need more effective data processing tools:**
  - Damage detection methods that work!
- **Physics-based versus data-driven methods:**
  - Data-driven approaches are beginning to emerge for data mining

IBM

Future Needs: Community Resiliency

- **Response of urban center to natural and man-made hazards:**
  - Understanding of interconnectivity of infrastructure:
    - Infrastructure inventory kept by both public and private stakeholders
  - "Urban genome" project should be initiated:
    - Automation of assembling a complete inventory of regional infrastructure including points of connectivity
    - Regional-scale simulation to simulate infrastructure response
  - Societal resiliency highly dependent on human response:
    - Social science models that model human reactions to catastrophes

Non-traditional "Sensor" Data Streams?

Community Sourced Data with Geospatial Mapping?

Thank You!

Acknowledgements:

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Michigan Department of Transportation (MDOT)
California Department of Transportation (Caltrans)
National Science Foundation (NSF)
Smart Infra-Structure Technology Center (SISTEC)

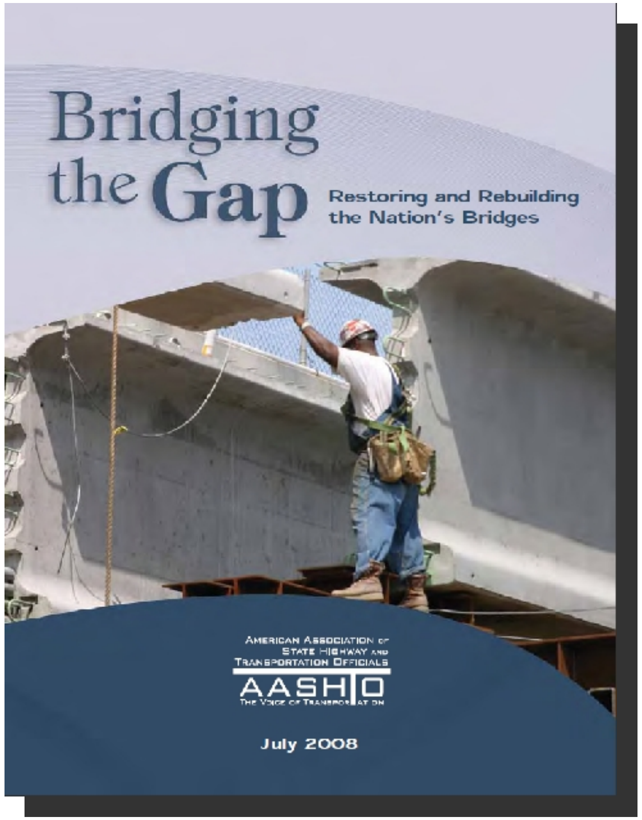
Critical National Resource: Bridges

George Washington Bridge, New York

- **In the United States, there are more than 600,000 bridges:**
  - These bridges fall within federal, state and local jurisdiction
  - 90% of U.S. truck traffic travels over state-owned bridges
- **Post-WWII construction period:**
  - Major expansion of bridge inventory mirrors post-WWII economic growth
  - Within the next 15 years, 50% of the nation's bridges will exceed 50 years
    - "Baby boomer" problem for the US
    - With age comes deterioration ($$)

AASHTO’s “Bridging the Gap”

- **5 major problems for US bridges:**
  - Age and deterioration
  - Bridges are chokepoints within the freeway system
  - Soaring construction costs means states must do less for more money
  - Delay in new bridge construction jeopardize economic growth
  - Unable to maintain bridge safety due to funding shortages



Bridging the Gap

Restoring and Rebuilding the Nation’s Bridges

American Association of State Highway and Transportation Officials

AASHTO

The Voice of Transportation

July 2008

- **Top 2 solutions for bridge systems:**
  - Investment - have to significantly increase transportation investment in the US
  - Research and Innovation - innovations in design, materials, and associated technology are needed to advance a new generation of safe and long-lasting bridges

NIST Technology Innovation Program

- **National Institute of Standards and Technology (NIST) Technology Innovation Program (TIP):**
  - 5-year project focused on advancing SHM systems for bridges
  - Address the aforementioned limitations of existing SHM systems
- **Comprehensive re-design of bridge SHM systems:**
  - Based on wireless telemetry as a core building block of the system
  - Researchers have emphasized sensors aimed at getting better data
  - We are focused on getting end-users “information,” not just “data”

MICHIGAN

University of Michigan (LEAD)

Li, Fischer, Lepech & Associates

Prospect Solutions, LLC

WEIDLINGER ASSOCIATES® INC
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NIST

Caltrans

MDOT

Government Partners

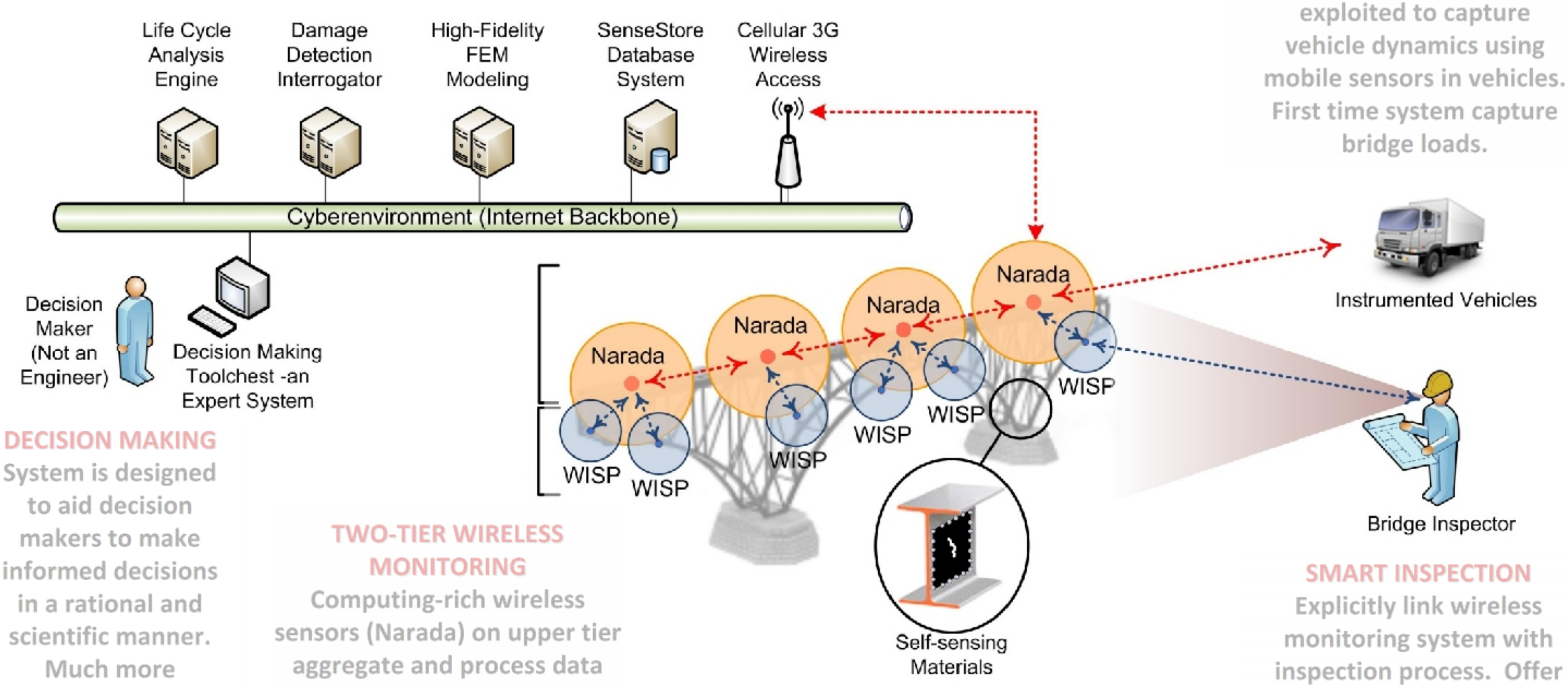
NIST TIP Project Overview

CYBERINFRASTRUCTURE

Data is passed to the internet by 3G cellular network where it is stored in a database and analyzed using various data mining tools

VEHICLE INTERACTION

Wireless communication exploited to capture vehicle dynamics using mobile sensors in vehicles. First time system capture bridge loads.



DECISION MAKING

System is designed to aid decision makers to make informed decisions in a rational and scientific manner. Much more effective than inundating owners with raw data

TWO-TIER WIRELESS MONITORING

Computing-rich wireless sensors (Narada) on upper tier aggregate and process data from low-power wireless slaves (WISP) on lower tier. Wireless sensing saves cost by one order of magnitude.

SELF-SENSING MATERIALS

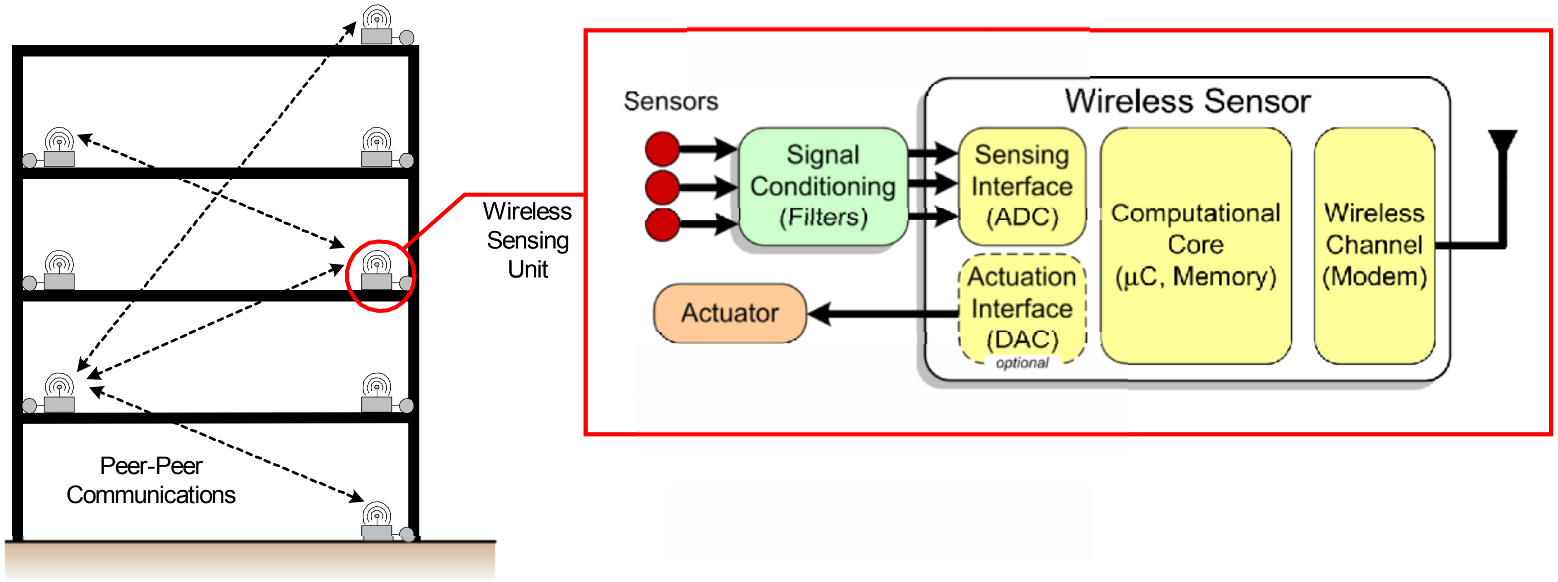
Self-sensing materials including ECC and CNT sensing skins provide detailed, local information on structural damage and degradation *directly*.

SMART INSPECTION

Explicitly link wireless monitoring system with inspection process. Offer modes of interaction between inspector and bridge.

Wireless Structural Monitoring

- **Wireless sensing proposed by Straser & Kiremidjian (1996)**
- **Wireless sensor networks are today viable substitutes:**
  - System constructed from low-cost wireless sensors (~$100 per node)
  - Low cost drives *high-density installation* targeting local damage
  - Computational power is coupled with sensors for data interrogation



Architectural design of wireless structural monitoring systems

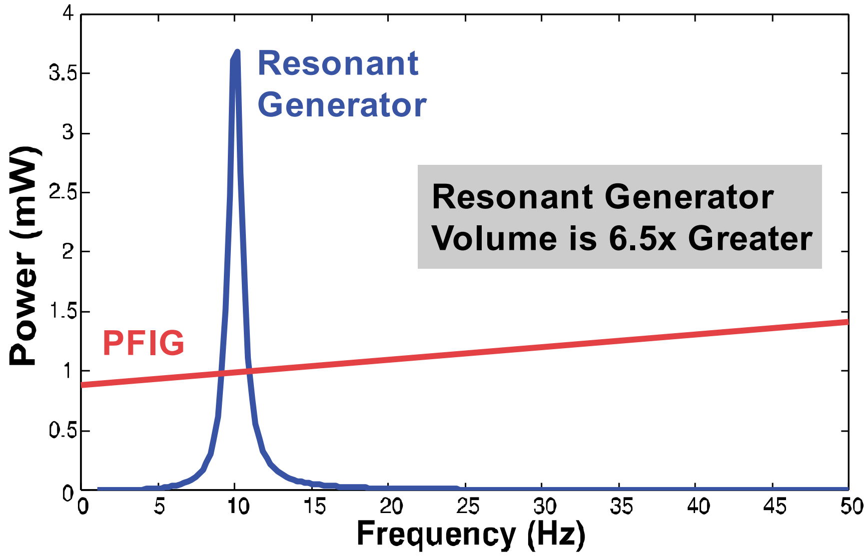
Power Challenges

- **Power remains the Achilles heel of wireless sensors:**
  - Approach #1 – lower power electronics:
    - Power consumption continues to reduce in the microcontroller and sensor marketplaces
  - Approach #2 - power harvesting:
    - Solar panels used to keep *Narada* battery pack charged
    - Ambient vibrations targeting powering wireless sensor nodes

Battery replacement a management headache and environmental challenge

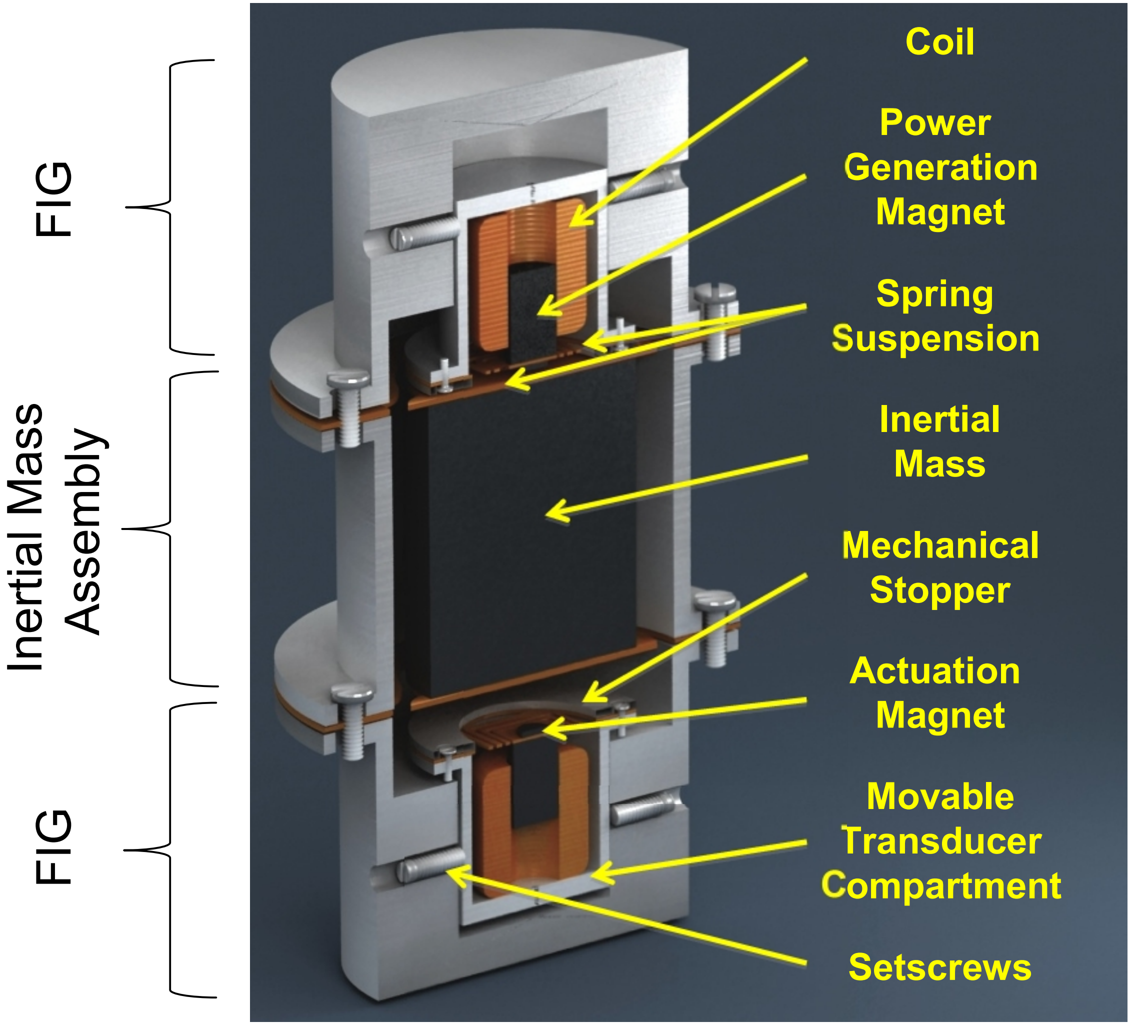
Harvesting Power from Vibrations

- **Challenges with mechanical harvesting on bridges:**
  - Low frequency (<10 Hz) non-periodic vibrations
  - Low forces available (< 0.1 g of response acceleration) requiring mass
- **Parametric Frequency-Increased Generator (PFIG):**
  - Offers large bandwidth (22 Hz)
  - Requires minimum input acceleration for actuation



In collaboration with Prof. Khalil Najafi and Dr. Becky Peterson (Michigan)

4th Generation PFIG Design



FIG

Inertial Mass Assembly

FIG

Coil

Power Generation Magnet

Spring Suspension

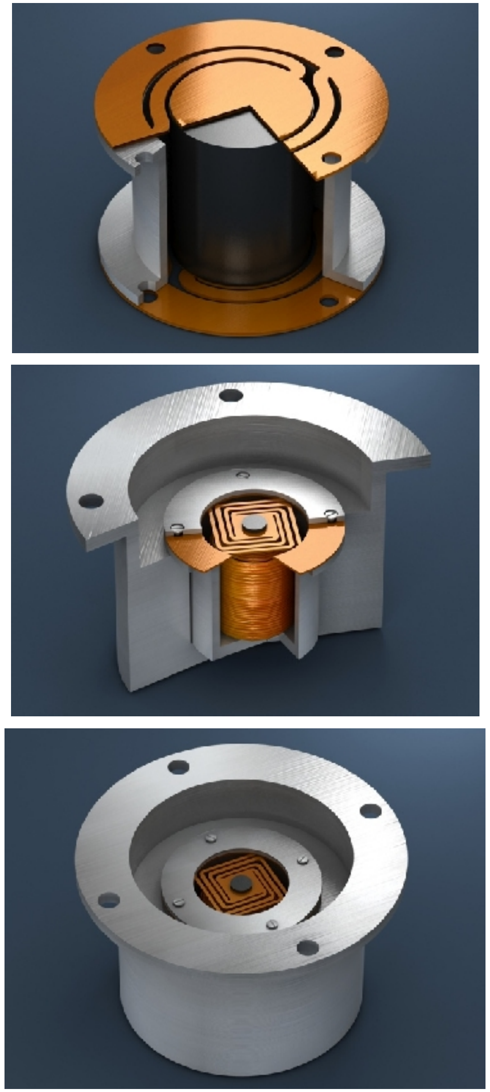
Inertial Mass

Mechanical Stopper

Actuation Magnet

Movable Transducer Compartment

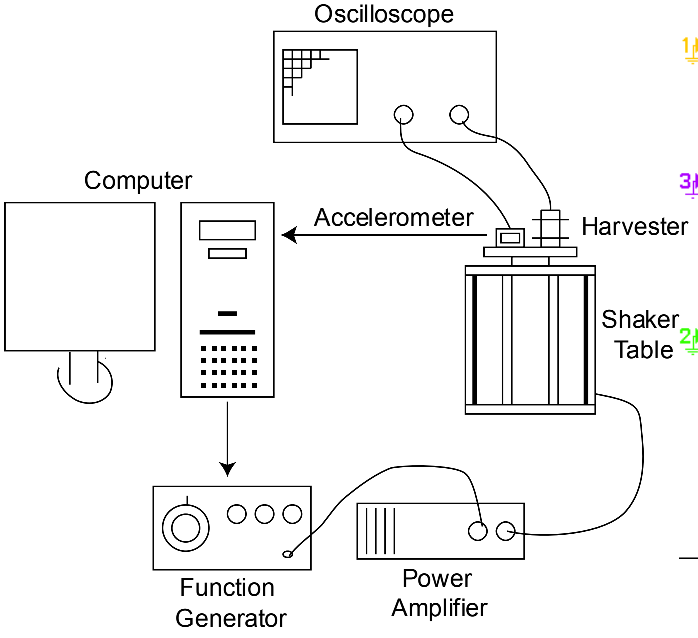
Setscrews



PFIG Performance

Input Vibration

0.055 g @ 2Hz



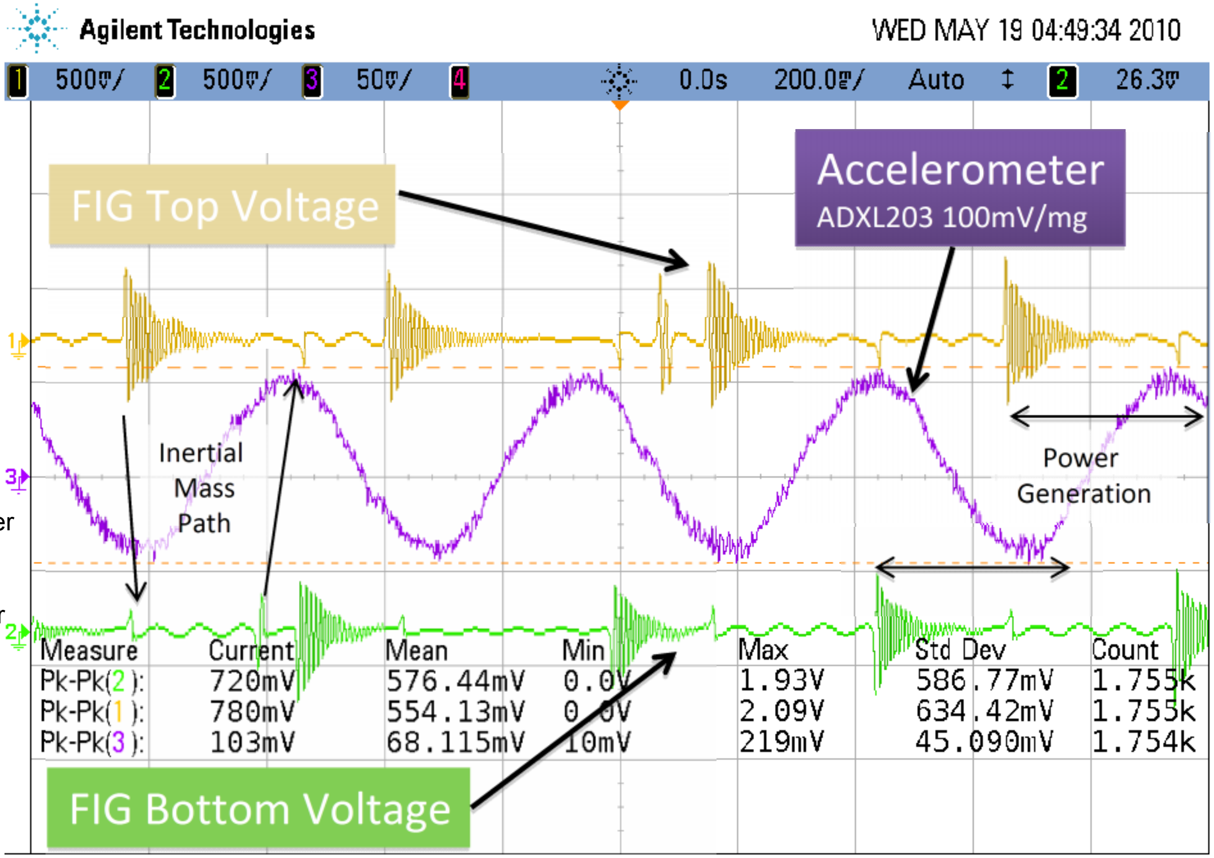


FIG Top Voltage

Accelerometer
ADXL203 100mV/mg

Inertial Mass Path

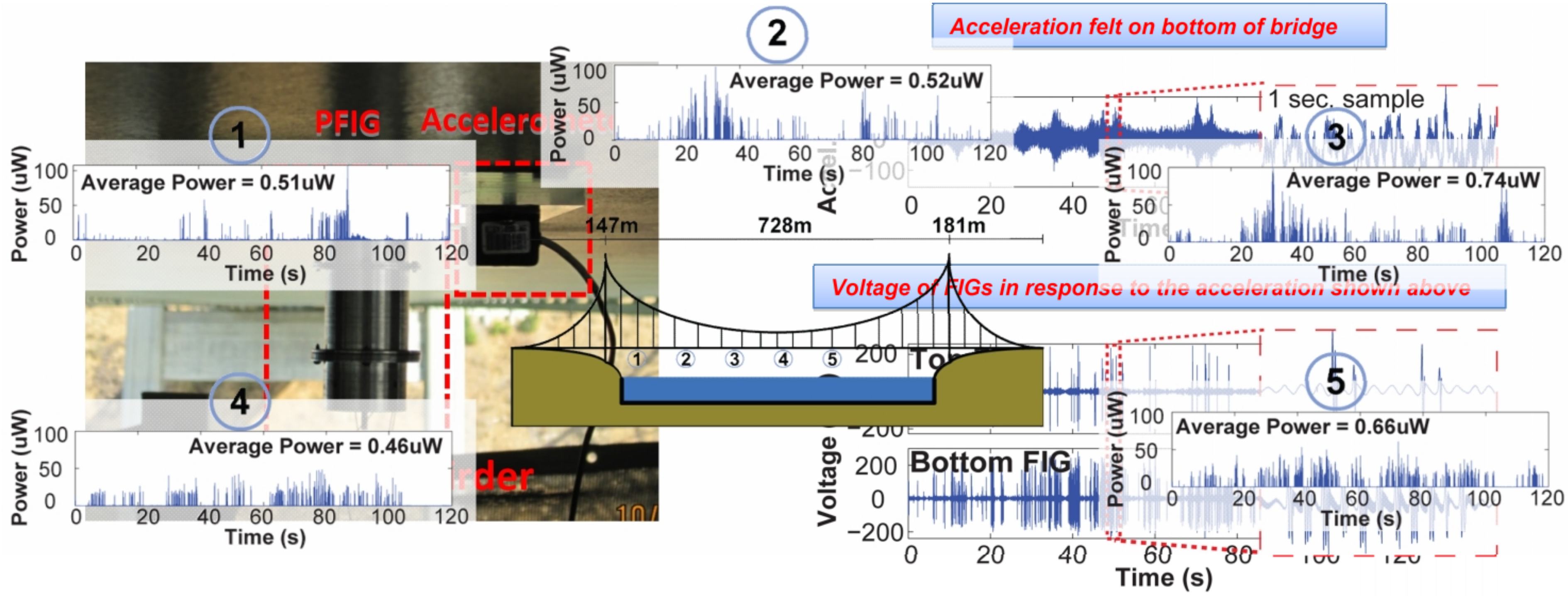
Power Generation

Measure	Current	Mean	Min	Max	Std Dev	Count
Pk-Pk(2):	720mV	576.44mV	0.0V	1.93V	586.77mV	1.755k
Pk-Pk(1):	780mV	554.13mV	0.0V	2.09V	634.42mV	1.755k
Pk-Pk(3):	103mV	68.115mV	10mV	219mV	45.090mV	1.754k

FIG Bottom Voltage

Power Harvesting on NCB

- **PFIG validated on the NCB during ambient vibrations:**
  - PFIG closely monitored using a LabView DAQ system
  - Accelerations in the 30 - 100mg range experienced
  - 0.5-0.75uW constant (average) supply capability verfieid



Optimization

- **Identification of B is an unconstrained optimization problem:**

Measured bridge response

$$\{\hat{\mathbf{B}}(0),\hat{\mathbf{B}}(1),\cdots,\hat{\mathbf{B}}(N-1)\}=\underset{\{\mathbf{B}(0),\mathbf{B}(1),\cdots,\mathbf{B}(N-1)\}}{\arg\min}\frac{1}{N}\sum_{k=1}^{N}\left\|\mathbf{y}(k)-\hat{\mathbf{y}}(k\mid k-1)\right\|^2$$

Predicted bridge response

Nonlinear search problem
along time axis
(a very challenging
problem !!)

$$\hat{\mathbf{y}}(k\mid k-1)=\hat{\mathbf{C}}\,\hat{\mathbf{A}}^{k}\,\mathbf{x}(0)+\sum_{q=1}^{k}\hat{\mathbf{C}}\,\hat{\mathbf{A}}^{k-q}\,\mathbf{B}(k-1)u(q)$$

- **Kernel approximation:**
  - Identified input location by a local basis vector

$$\mathbf{B}(k):=\alpha(k)\boldsymbol{\Phi}(k)\qquad\boldsymbol{\Phi}(k)=\mathbf{L}\boldsymbol{\Phi}_c(t)\qquad\mathbf{L}:=\left(\hat{\mathbf{A}}-\mathbf{I}\right)\left(\frac{1}{\Delta t}\ln\hat{\mathbf{A}}\right)^{-1}\in\mathrm{R}^{n\times n}$$

Dynamic weighting factor

Time-variant Kernel (local basis vector)

Location vector

Linear mapping operator

$$\{\hat{\alpha}(0),\hat{\alpha}(1),\cdots,\hat{\alpha}(N-1)\}=\underset{\{\alpha(0),\alpha(1),\cdots,\alpha(N-1)\}}{\arg\min}\frac{1}{N}\sum_{k=1}^{N}\left\|\mathbf{y}(k)-\sum_{q=1}^{k}u(q)\hat{\mathbf{C}}\,\hat{\mathbf{A}}^{k-q}\,\boldsymbol{\Phi}(q)\alpha(k-1)\right\|^2$$

$$\{\hat{\mathbf{B}}(0),\hat{\mathbf{B}}(1),\cdots,\hat{\mathbf{B}}(N-1)\}=\{\hat{\alpha}(0)\boldsymbol{\Phi}(0),\hat{\alpha}(1)\boldsymbol{\Phi}(1),\cdots,\hat{\alpha}(N-1)\boldsymbol{\Phi}(N-1)\}$$