Future Infrastructure Forum

Cambridge, January 2012

Steve Denton

Director of Engineering, Parsons Brinckerhoff

Visiting Professor, University of Bath

PARSONS BRINCKERHOFF

The brief...

…try to identify … where you think research funds should be directed to help address the pressing challenges facing society

The brief...

…try to identify … where you think research funds should be directed to help address the pressing challenges facing society

An unprecedented series of challenges...

- Carbon reduction targets
- Long term energy security
- Growing demand on ageing networks
- Climate change
- Increasing interdependencies
- Overstretched public finances

National Infrastructure Plan, 2010

… for several decades the UK's approach to infrastructure has in general been timid, uncoordinated, incremental, wasteful in its procurement and insufficiently targeted …

National Infrastructure Plan, 2010

Cause for concern?

- World economic forum rated UK 33rd for the overall quality of our infrastructure in 2010

Cause for concern?

- World economic forum rated UK 33rd for the overall quality of our infrastructure in 2010
- … by 2011 we had progressed to 28th

The brief...

…try to identify … where you think research funds should be directed to help address the pressing challenges facing society

EPSRC Strategic Plan

PARSONS BRINCKERHOFF

OUR STRATEGY
FOR THE NEXT THREE
TO FIVE YEARS

EPSRC WILL DELIVER CHANGE THROUGH

THREE CLEAR GOALS

DELIVERING IMPACT

EPSRC will ensure excellent research and talented people deliver maximum impact for the health, prosperity and sustainability of the UK. We will build strong partnerships with organisations that can capitalise on our research and inform our direction. We will promote excellence and impact, and ensure it is visible to all.

SHAPING CAPABILITY

EPSRC will shape the research base to ensure it delivers high quality research for the UK, both now and in the future. Our research portfolio will be focused on the strategic needs of the nation, such as green technologies and high-value manufacturing, and will retain the capability to tackle future challenges and capitalise on new opportunities. We will set the bar high, stimulate creativity and reward ambition.

DEVELOPING LEADERS

We will commit greater support to the world-leading individuals who are delivering the highest quality research to meet UK and global priorities. We will create an environment that supports them throughout their careers and allows others to benefit from their ability. We will foster their ambition and adventure and ensure they are connected with the best, wherever and whoever they are in the world.

The brief...

…try to identify … where you think research funds should be directed to help address the pressing challenges facing society

PARSONS BRINCKERHOFF

STRATEGY

STRATEGY

↑

TACTICS

VISION

STRATEGY

TACTICS

What will be the same…

What will be the same…

2.1 Basic requirements

(2)P A structure shall be designed to have adequate :
- structural resistance,
- serviceability, and
- durability.

(4)P A structure shall be designed and executed in such a way that it will not be damaged by events such as :
- explosion,
- impact, and
- the consequences of human errors,

to an extent disproportionate to the original cause.

What will be different…

What will be different…

- New procurement paradigms
- Technology will be used to get the most from our infrastructure (smart infrastructure)
- Ageing infrastructure will be even older – time bomb?
- Better asset knowledge (BIM)
- Changing climate
- New materials

What will be different…

- New procurement paradigms
- Technology will be used to get the most from our infrastructure (smart infrastructure)
- Ageing infrastructure will be even older – time bomb?
- Better asset knowledge (BIM)
- Changing climate
- New materials

Research issues

- Interdependencies
- Robustness
- Resilience

What will be different…

- New procurement paradigms
- Technology will be used to get the most from our infrastructure (smart infrastructure)
- Ageing infrastructure will be even older – time bomb?
- Better asset knowledge (BIM)
- Changing climate
- New materials

Research issues

- How do we make the right decisions?

PARSONS BRINCKERHOFF

Research issues

- How do we make the right decisions?
- Need to challenge and refresh fundamental understanding and approaches

Paper

The assessment of reinforced concrete slabs

S. R. Denton, BA
Rust Consulting Ltd

C. J. Burgoyne, MA, MSc, PhD, CEng, MIStructE, MICE
University of Cambridge

Synopsis

In the design of reinforced concrete slabs the Wood-Armer equations are used extensively. However, their direct application to assessment can result in a conservative estimate of structural capacity. Equations based on the same fundamental principles are derived which provide a more precise measure of the ability of a given slab to withstand an imposed field of moments. Application of these equations will lead, in many cases, to an improved assessment for bridges previously analysed using the Wood-Armer equations and found to require a load restriction.

Introduction

The Wood-Armer equations were derived for the design of reinforced concrete slabs subject to complex loadings. The equations ensure that the capacity of a slab is not exceeded in flexure by an imposed loading, whilst minimising the total amount of reinforcement required. However, the use of these equations for assessment leads to a conservative estimate of structural capacity in all cases where steel is not distributed optimally. The optimality condition is a constraint in design problems that is not relevant to assessment problems, and its use can lead to adequate structures being condemned as unsafe.

The present analysis is based on the same fundamental principles as those set out by Hillerbourg[1], which were extended by Wood[2] and Armer[3] in the derivation of the Wood-Armer equations, but it assumes that the reinforcement arrangement is already known. The methodology provides a systematic approach to assess whether a reinforced concrete slab has sufficient capacity to withstand an imposed loading, quantified by determination of the factor of safety on that loading.

Loading and capacity field equations

To maintain consistency with the Wood-Armer derivations the axis system used by Wood has been adopted and is shown in Fig 1. As a number of different conventions can be used to define bending moments it is worth emphasising that, in the following analysis, the applied bending moment M_x is about an axis perpendicular to the x-axis, so that it gives rise to stresses in the x-direction. The same convention is adopted for moments of resistance which are denoted by M^*. Thus, steel parallel to the x-axis contributes primarily to the capacity term M_x^*.

It will be observed that the convention used for moments of resistance differs slightly from that used by Wood, since the present method is concerned with analysis rather than design. Here, M_x^* is the total moment of resistance of the slab about an axis perpendicular to the x-axis, including any contribution made by reinforcement at a skew angle to the x-axis. Wood, on the other hand, used M_x^* to denote the moment of resistance needed from reinforcement parallel to the x-axis alone. For orthogonal reinforcement both conventions yield the same numerical values for M_x^* and M_y^*.

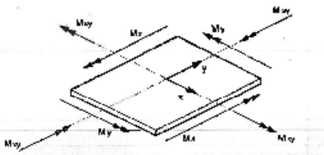


Fig 1. Notation for bending moments (positive as shown)

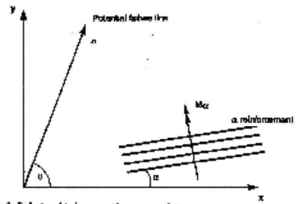


Fig 2. Relationship between the x-, n- and α-axes

For simplicity, moments are represented by the triad (M_x, M_y, M_{xy}). An asterisked triad $(\_\_, \_\_, \_\_)^*$ will indicate moment capacity.

As defined in Fig 1, hogging moments are positive, and thus require steel primarily in the top face. Steel will be needed in the bottom face to resist negative moments. Analogous equations can be derived for other sign conventions, for both the flexural moments M_x and M_y and the twisting moment M_{xy}.

All moments in the analysis which follows will be expressed as moments/unit length, so will have units of force. It will be assumed that all sections are significantly underreinforced, so steel in the bottom face of the slab will affect only the sagging moment of resistance and will have no influence on the hogging moment of resistance, since it is adding to a compressive strength that is already more than adequate.

The flexural load effects at a point in a plane slab due to an imposed loading are fully defined by the moment triad (M_x, M_y, M_{xy}). The bending moment M_n, about any other axis (see Fig 2), can be derived solely by equilibrium, giving:

$$M_n = M_x \cos^2\theta + M_y \sin^2\theta - 2M_{xy}\sin\theta\cos\theta \qquad ....(1)$$

For a single layer of reinforcement at an angle of α, as shown in Fig 2, the moment of resistance about the normal to the n-axis, M_n^*, calculated by applying Johansen's stepped criterion of yield[4], is given by

$$M_n^* = M_\alpha^* \cos^2(\theta-\alpha) \qquad ....(2)$$

The $\cos^2$ function accounts for the effective increase in steel spacing across a skew hinge and the reduced component of steel stress acting perpendicular to the hinge. This equation has been verified experimentally (Morley[5]).

Eqn. (2) may be rewritten as

$$M_n^* = M_x^* \cos^2\theta + M_y^* \sin^2\theta - 2M_{xy}^* \sin\theta\cos\theta \qquad ....(3)$$

where

$$M_x^* = M_\alpha^* \cos^2\alpha \qquad ....(3a)$$
$$M_y^* = M_\alpha^* \sin^2\alpha \qquad ....(3b)$$
$$M_{xy}^* = -M_\alpha^* \cos\alpha\sin\alpha \qquad ....(3c)$$

It is a reasonable approximation to assume that multiple layers of reinforcement with different orientation act independently, although this is not strictly the case since the interaction of skewed layers of steel slightly alters the neutral axis depth.

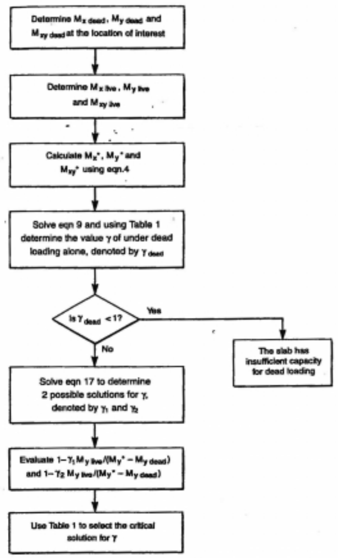


Fig 8. Procedure for assessment under combined dead and live loading

From Table 1, this can be identified as case 2, so the safety factor on the applied loading is γ_2 (=1.346).

For the same applied moment field, the reinforcement moments calculated using the Wood-Armer equations are 60.7 kNm/m for the reinforcement parallel to the x-axis and 33.4 kNm/m for the skew reinforcement.

If the value of the skew reinforcement moment calculated using the Wood-Armer equations is compared with the actual moment of resistance of the skew reinforcement alone, the resulting factor of safety is 35/33.4 (= 1.04). Thus an improvement in the assessed capacity of approximately 30% is achieved through the use of the present approach in this case.

Assessment under dead and live loading

When a slab is assessed to determine whether it has sufficient capacity to withstand some additional loading or when a slab is subjected to a combination of dead and live loading, it is often more informative to calculate the factor of safety on the live (or additional) loading after the full dead (or permanent) loading has been applied. This assessment requires two stages – the first to ensure that the structure can withstand the dead load and, if it passes that test, a second analysis to see how much live load can be carried.

The first analysis can be undertaken by the method given above, but a modification is required for the second analysis. In this case, the dead load moments have to be taken into account. This can be done by subtracting the dead load moments from the load capacity, to give the load capacity available for live load moments.

Thus, $M_n^*{}_{live} = M_n^* - M_{n\,dead}$

Although this is the principle of the revised analysis, it is convenient not to have to calculate the live load capacities directly. Instead, an approach similar to that used above can be developed. It then follows that, at the critical angle θ_o,

$$M_n^* - M_{n\,dead} - \gamma M_{n\,live} = \left(M_x^* - M_{x\,dead} - \gamma M_{x\,live}\right)\cos^2\theta_o + \left(M_y^* - M_{y\,dead} - \gamma M_{y\,live}\right)\sin^2\theta_o - 2\left(M_{xy}^* - M_{xy\,dead} - M_{xy\,live}\right)\cos\theta_o\sin\theta_o = 0 \qquad ....(15)$$

and

$$\frac{d\left(M_n^* - M_{n\,dead} - \gamma M_{n\,live}\right)}{d\theta} = 0 \qquad ....(16)$$

where $M_{x\,dead}$, $M_{y\,dead}$ and $M_{xy\,dead}$ define the dead or permanent loading field and $M_{x\,live}$, $M_{y\,live}$ and $M_{xy\,live}$ define the live or additional loading field.

As before, these conditions can be rearranged to give a quadratic in γ

$$\left\{M_{x\,live}M_{y\,live} - \left(M_{xy\,live}\right)^2\right\}\gamma^2 + \left\{\begin{array}{l}2M_{xy\,live}\left(M_{xy}^* - M_{xy\,dead}\right) - M_{y\,live}\left(M_x^* - M_{x\,dead}\right)\\ -M_{x\,live}\left(M_y^* - M_{y\,dead}\right)\end{array}\right\}\gamma + \left\{\left(M_x^* - M_{x\,dead}\right)\left(M_y^* - M_{y\,dead}\right) - \left(M_{xy}^* - M_{xy\,dead}\right)^2\right\} = 0 \qquad ....(17)$$

which has two solutions γ_1 and γ_2. The criterion for selecting the correct value for γ is similar to that for a single applied moment field and is governed by the equation,

$$1 - \gamma\left(M_{y\,live}/\left(M_y^* - M_{y\,dead}\right)\right) \geq 0 \qquad ....(18)$$

Table 1 may be used to identify the required value of γ, and the procedure for assessing a slab for live loading is shown in Fig 8.

Example 2

Suppose that the loads applied to the slab in example 1 represented the dead loading, so that

M_{dead} = (35, 15, 10) kNm/m

and that the live loads, also determined by a suitable (but here unspecified) analysis technique, are

M_{live} = (6, 4, 5) kNm/m.

It was shown in the first example that the safety factor was greater than one; the slab therefore has some capacity available for live loads.

From the solution of eqn. (17), it follows that

$\gamma_1 = -585.4$
$\gamma_2 = 1.106$

so that,

$1 - \gamma_1(M_{y\,live}/(M_y^* - M_{y\,dead})) = 148.22$
$1 - \gamma_2(M_{y\,live}/(M_y^* - M_{y\,dead})) = 0.722$

From Table 1, this can be identified as case 4, so the safety factor on the applied loading γ_2 (= 1.106). The slab therefore has sufficient capacity to withstand the combined live and dead loading.

For the same applied moment fields, the reinforcement moments calculated for the skew reinforcement using the Wood-Armer equations are 33.4 kNm/m for the dead load alone and an additional 11.4 kNm/m when the live load is added. If these values are compared with the actual skew reinforcement capacity, the resulting factor of safety on live loading is (35-33.4)/11.4 (= 0.14), which is clearly inadequate. Thus, whilst the use of the Wood-Armer equations suggests that the slab only has sufficient capacity to withstand 14% of the live loading in combination with the dead loading, the present analysis demonstrates that the slab can withstand the full combined loading. There would be no need to take remedial action for this slab.

Conclusions

The Wood-Armer equations, originally derived for design purposes, provide a conservative assessment of the capacity of a reinforced concrete slab because of their use of an optimality condition. However, by adopting the

OUR STRATEGY
FOR THE NEXT THREE
TO FIVE YEARS

EPSRC WILL DELIVER CHANGE THROUGH

THREE CLEAR GOALS

DELIVERING IMPACT

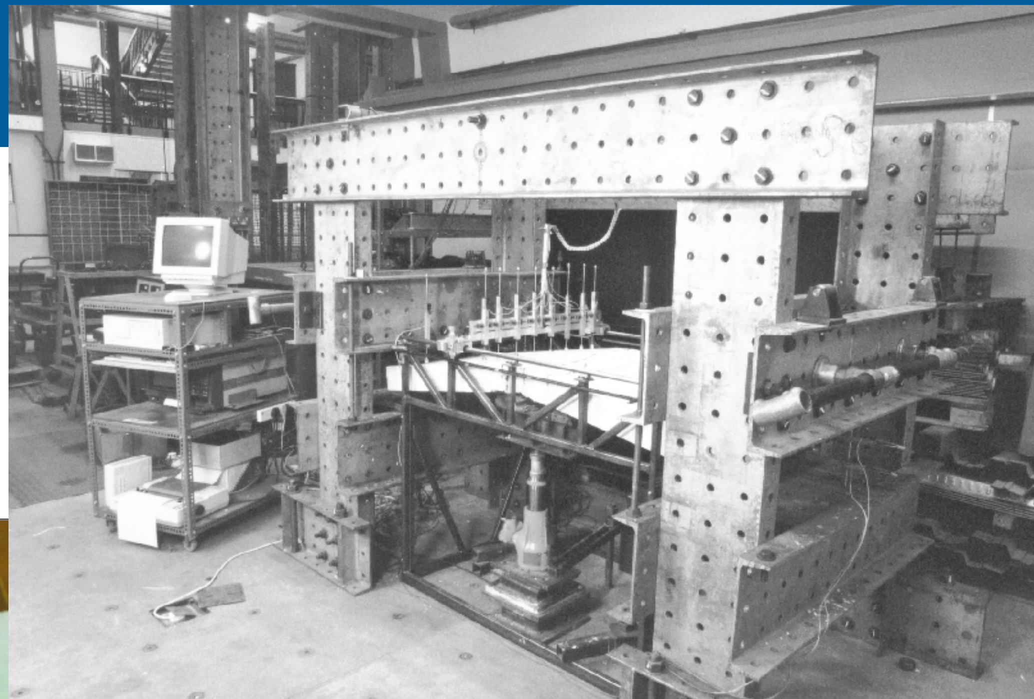
EPSRC will ensure excellent research and talented people deliver maximum impact for the health, prosperity and sustainability of the UK. We will build strong partnerships with organisations that can capitalise on our research and inform our direction. We will promote excellence and impact, and ensure it is visible to all.

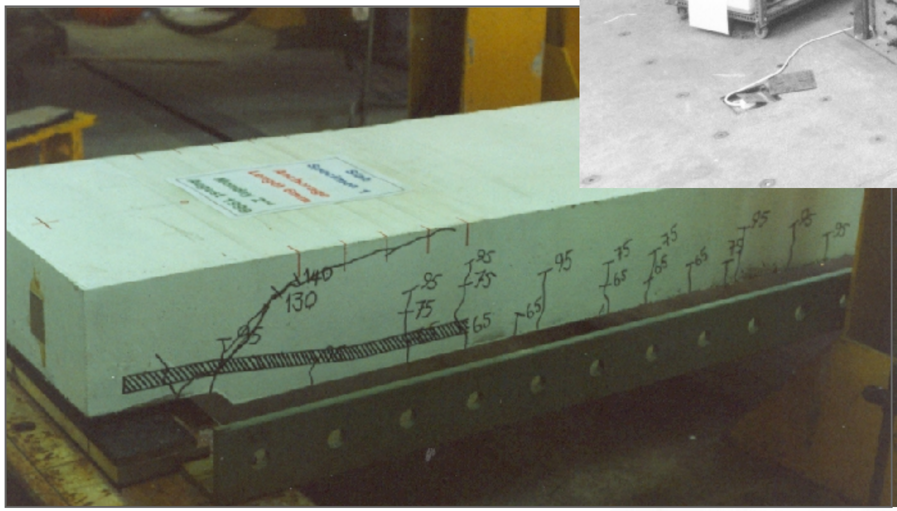
SHAPING CAPABILITY

EPSRC will shape the research base to ensure it delivers high quality research for the UK, both now and in the future. Our research portfolio will be focused on the strategic needs of the nation, such as green technologies and high-value manufacturing, and will retain the capability to tackle future challenges and capitalise on new opportunities. We will set the bar high, stimulate creativity and reward ambition.

DEVELOPING LEADERS

We will commit greater support to the world-leading individuals who are delivering the highest quality research to meet UK and global priorities. We will create an environment that supports them throughout their careers and allows others to benefit from their ability. We will foster their ambition and adventure and ensure they are connected with the best, wherever and whoever they are in the world.





PARSONS BRINCKERHOFF

Concrete design strength

$$f_{cd} = \alpha_{cc} \frac{f_{ck}}{\gamma_M}$$

Eurocode approach (EN 1990)

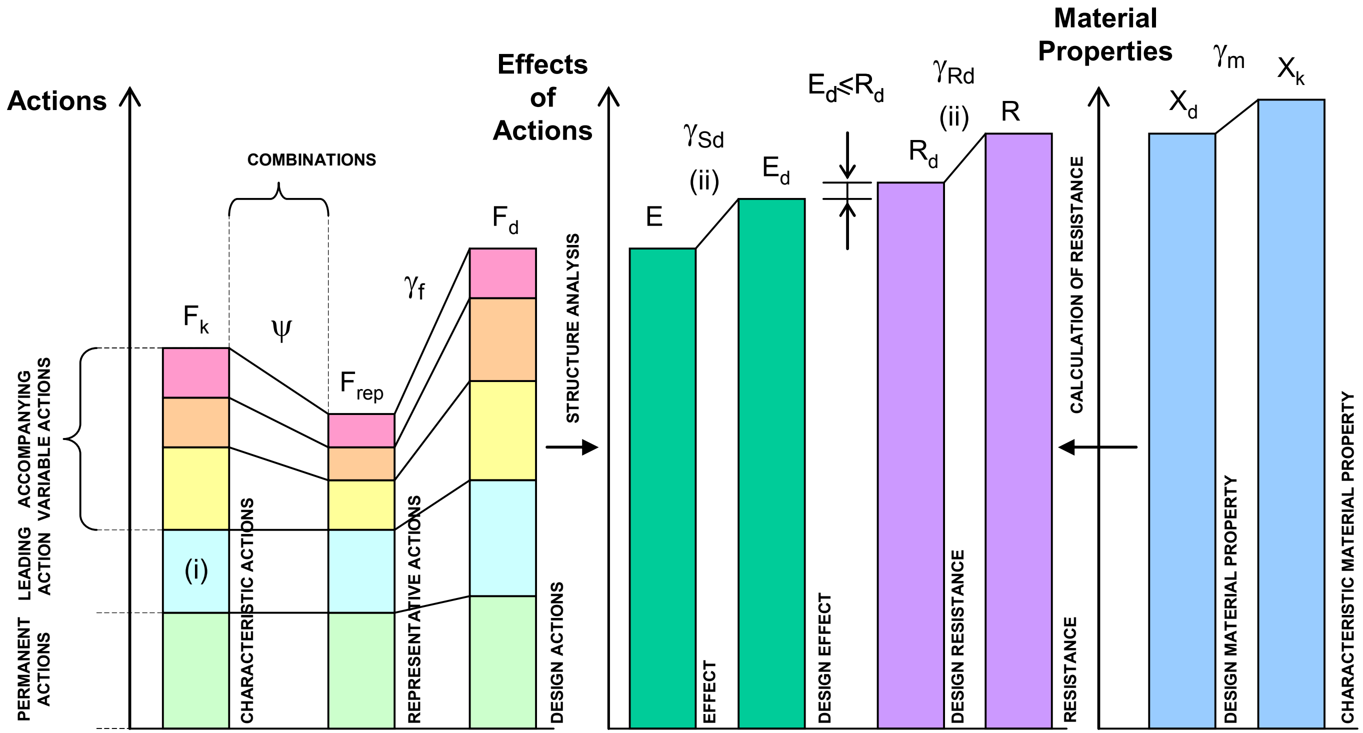
$$X_d = \eta \frac{X_k}{\gamma_m}$$

Eurocode approach (EN 1990)

$$X_d = \eta \frac{X_k}{\gamma_m}$$

Conversion factor

Eurocode approach (EN 1990)



ULS

(i) Where the action is a traffic load group, ψ factors will have been pre-applied to the non-leading actions within that group

(ii) In many cases, γ_{Sd} may be combined with γ_f and applied as a single factor γ_F to the actions, and γ_{Rd} is combined with γ_m and applied as a single factor γ_M to the material properties.

Typical Eurocode approach (EN 1990)

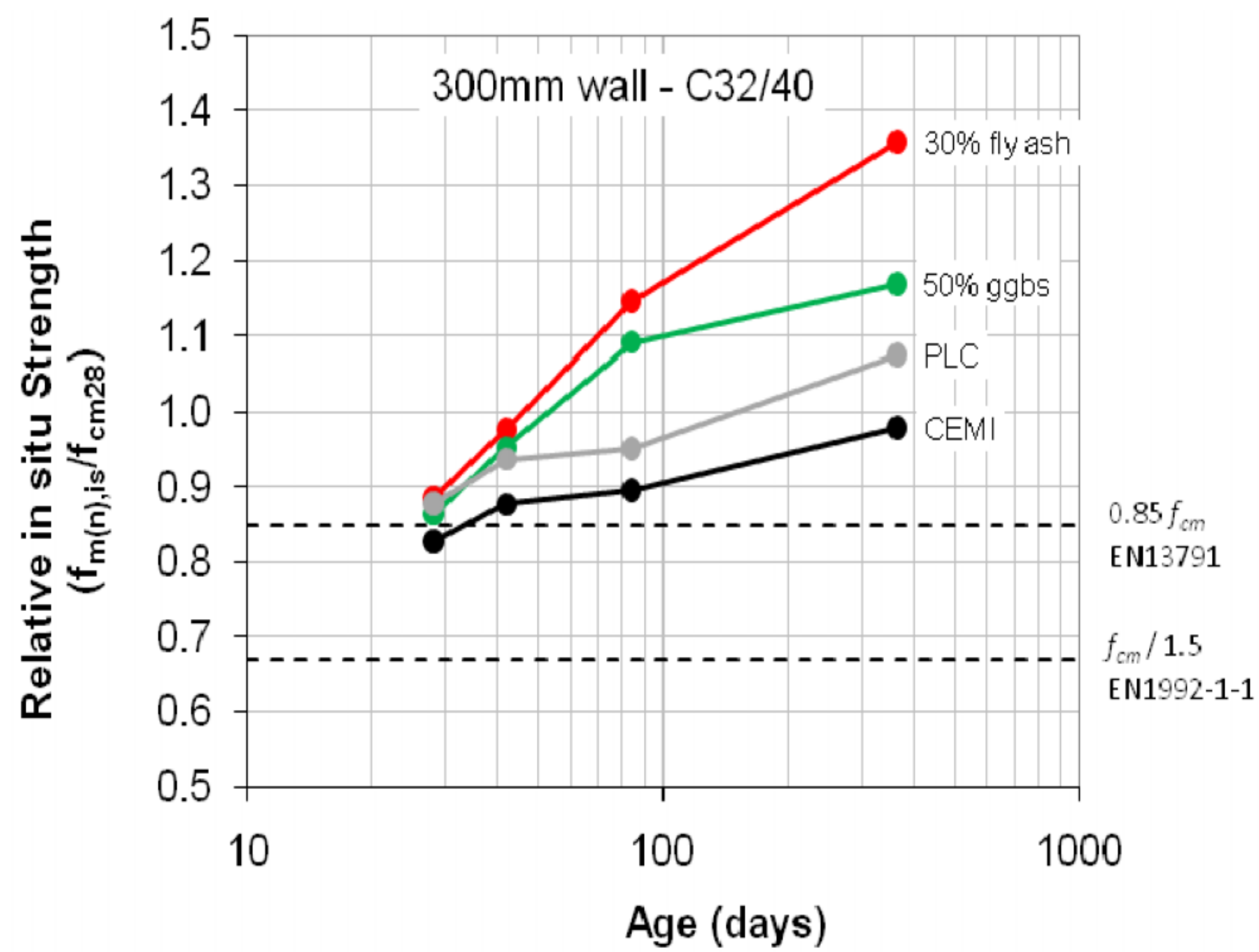
$$X_d = \frac{X_k}{\gamma_M}$$

Includes:
Conversion factor, η
Model factor, γ_{Rd}

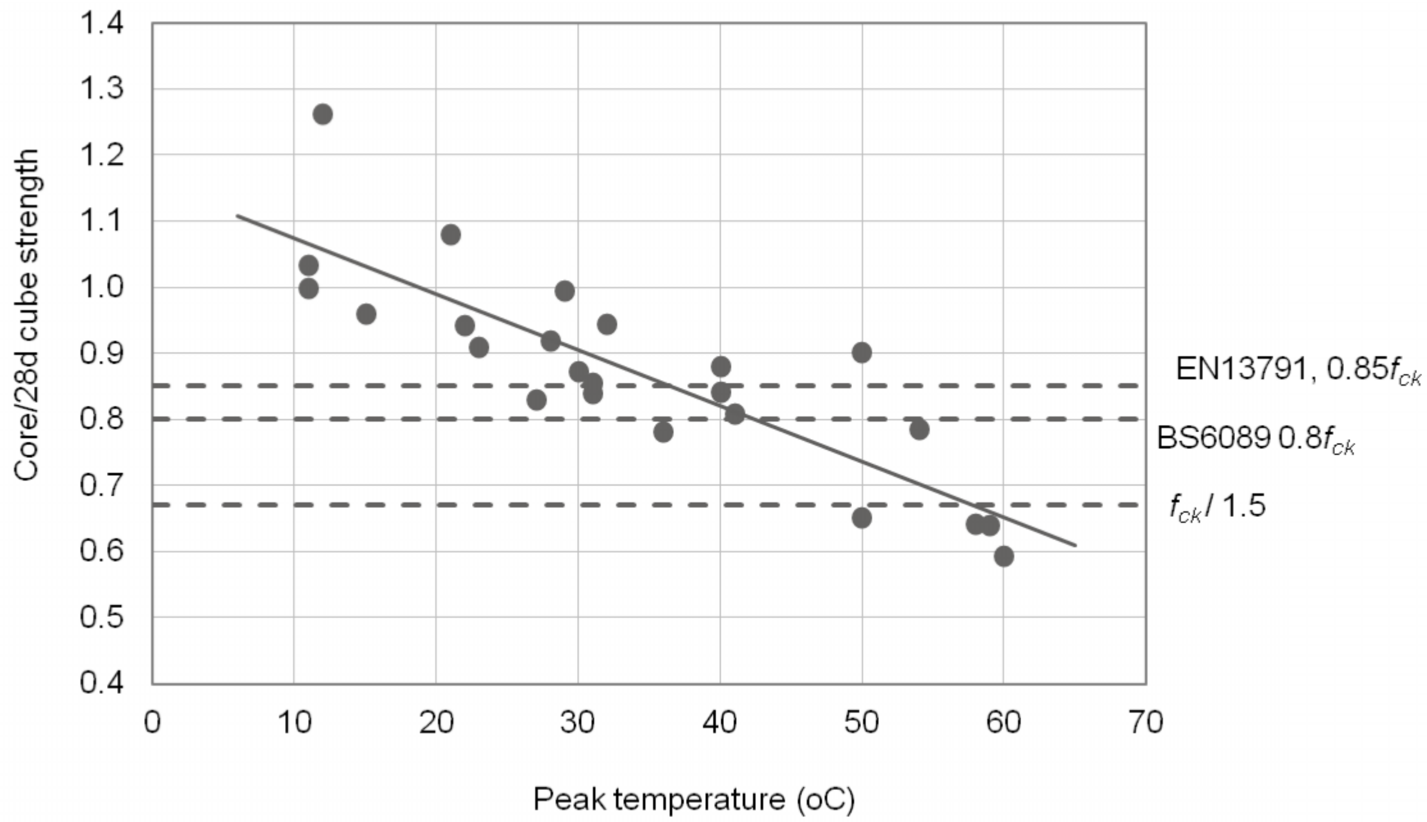
Concrete design strength

$$f_{cd} = \alpha_{cc} \frac{f_{ck}}{\gamma_M}$$

PARSONS BRINCKERHOFF

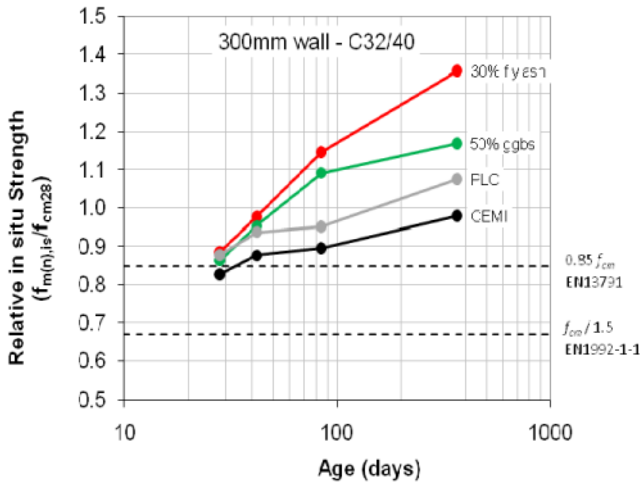


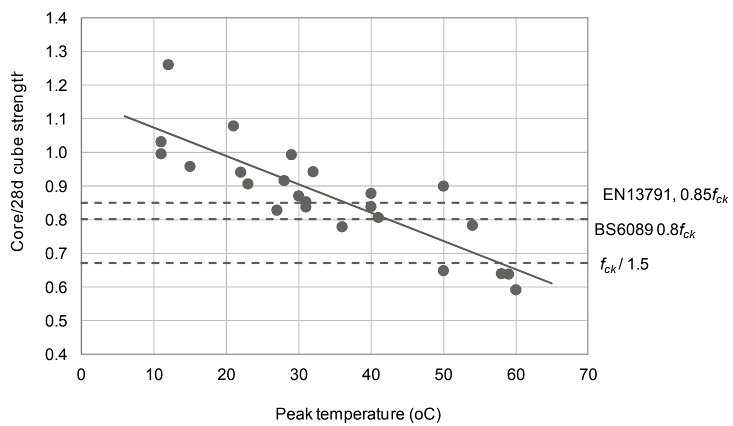
300mm wall - C32/40
Relative in situ Strength
($f_{m(n),is}/f_{cm28}$)
Age (days)
30% fly ash
50% ggbs
PLC
CEMI
$0.85\,f_{cm}$
EN13791
$f_{cm}\,/\,1.5$
EN1992-1-1
1.5
1.4
1.3
1.2
1.1
1.0
0.9
0.8
0.7
0.6
0.5
10
100
1000


1.4
1.3
1.2
1.1
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0
10
20
30
40
50
60
70
Core/28d cube strength
Peak temperature (oC)
EN13791, 0.85f_{ck}
BS6089 0.8f_{ck}
f_{ck}/ 1.5

Concrete design strength

$$f_{cd} = \alpha_{cc} \frac{f_{ck}}{\gamma_M}$$





Other issues

- Loading
- Soil-structure interaction
- …

Conclusions

- Challenges
- Funding goals
- Vision
- Understanding of needs and opportunities
- Fundamental re-examination